NTNU

Department of Mathematical Sciences

Examination paper for **ST0103 Statistics with Applications**

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Examination date: 7 December 2013
Examination time (from–to): 9:00–13:00
Permitted examination support material: Yellow A4 sheet with your own handwritten notes, specific basic calculator, *Tabeller og formler i statistikk* (Tapir forlag), *Matematisk formelsamling* (K. Rottmann)

Other information:
In the grading, each of the ten points counts equally.

You should demonstrate how you arrive at your answers (e.g. by including intermediate answers or by referring to theory or examples from the reading list).

Language: English
Number of pages: 1
Number pages enclosed: 0

Checked by:

Date Signature

Problem 1 A chemical reaction has velocity (measured in micromoles per hour) that has a normal distribution with expected value μ and known standard deviation $\sigma = 1.8$.

a) Assume (only here) that $\mu = 11$. What is the probability that the velocity is greater than 13? What is the conditional probability that the velocity is greater than 13 given that it is greater than 11?

The reaction was run $n = 15$ times, and the average velocity was $\bar{x} = 10{,}2$.

b) Find a 99 % confidence interval for the expected velocity μ. How large would n have to be for a 99 % confidence interval for μ to have a length less than 2?

c) The laboratory has as an aim to achieve an expected velocity of 11.0 micromoles per hour. Perform a hypothesis test with $\mu \geq 11.0$ as the null hypothesis and $\mu < 11.0$ as the alternative hypothesis. Use significance level 0.05.

d) What is the probability that the null hypothesis is rejected if we perform an experiment and a hypothesis test as described above, if $\mu = 10.2$?

Problem 2 A random variable X has probability density function f given by $f(x) = \theta x^{\theta-1}$ for $0 < x < 1$ and $f(x) = 0$ for all other x, where $\theta > 0$ is a parameter.

a) Find the expected value and variance of X.

b) Show that the cumulative distribution function F of X is given by $F(x) = x^\theta$ when $0 < x < 1$. Find $P(X > \frac{1}{2})$ in terms of θ.

c) Find the maximum likelihood estimator of θ based on n independent variables $X_1, X_2, \ldots, X_n$, all having probability density function f.

d) Find the cumulative distribution function and the probability density function of X^θ.

Problem 3 A biologist is to estimate the number of seal pups in a population. She finds X_1 pups. Assume that the probability p_1 of observing a pup is known, and that X_1 has a binomial distribution with parameters (n, p_1). We wish to estimate n.

a) Show that X_1/p_1 is an unbiased estimator of n. Find the variance of the estimator in terms of n and p_1. What is the estimate if $p_1 = 0.60$ and $X_1 = 150$?

The biologist performs another count using another method, and get the result X_2. Assume that X_2 has a binomial distribution with parameters (n, p_2), and that X_1 and X_2 are independent. It is known that $p_1 = 0.60$ and $p_2 = 0.71$, and n is the same in both counts.

b) Which of the two estimators $\frac{1}{2}(X_1/p_1 + X_2/p_2)$ and $0{,}38X_1/p_1 + 0{,}62X_2/p_2$ would you prefer? What are the two estimates if $X_1 = 150$ and $X_2 = 180$?