NTNU – Trondheim
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Department of Mathematical Sciences

Examination paper for **ST0103 Statistics with Applications**

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Examination date: 10 December 2014

Examination time (from–to): 9:00–13:00

Permitted examination support material: Yellow A4 sheet with your own handwritten notes, specific basic calculator, *Tabeller og formler i statistikk* (Tapir forlag), *Matematisk formelsamling* (K. Rottmann)

Other information:
In the grading, each of the ten points counts equally.

You should demonstrate how you arrive at your answers (e.g. by including intermediate answers or by referring to theory or examples from the reading list).

Language: English

Number of pages: 2

Number pages enclosed: 0

Checked by:

Date Signature

Problem 1

The flowers of a flower species, A, are circular with a radius that has a normal distribution with expected value 15 mm and standard deviation 2 mm. A botanist is collecting flowers of this species.

a) Find the probability that a flower that the botanist picks has a radius that is less than 17 mm. The botanist picks 10 flowers. The radii of the flowers are independent. Find the probability that 8 or more of them have radius less than 17 mm.

b) Which probability distribution does the circumference of a flower have (the circumference is $2\pi X$, where X is the radius)? Determine the parameters of the probability distribution.

A belongs to a genus having only two species. The other species, B, has circular flowers, too. The botanist is of the opinion that the flowers of B generally are somewhat larger, and measures 20 flowers of species B. She finds that the average radius of the 20 flowers is 19.0 mm. The sample standard deviation is 3.0 mm. Assume that the radius of the flowers of species B has a normal distribution with expected value μ (measured in mm) and unknown standard deviation.

c) Find a 99 % confidence interval for μ.

d) Perform the hypothesis test $H_0\colon \mu \leq 17$ against the alternative $H_1\colon \mu > 17$. Use significance level 0.05.

Assume for the remainder of the problem that it is known that the radius of a flower of species B has a normal distribution with expected value 19 mm and standard deviation 3 mm. The probability that a flower picked by the botanist is of species A is 0.4 and that it is of species B 0.6.

e) What is the probability that a flower picked by the botanist has radius less than 17 mm? What is the probability that a flower picked by the botanist is of species A if it has radius less than 17 mm?

The botanist wish to make a preliminary determination of species by defining flowers having radius less than c mm as species A and the remainder as species B.

f) What is the probability that a flower picked by the botanist is determined to belong to the wrong species if $c = 17$? Hos should c be chosen (between 15 and 19) for this probability to be as small as possible? (The last question is strenuous.)

Problem 2

The eruptions of a volcano follow a Poisson process having intensity λ (number of eruptions per year).

a) Assume (only here) that $\lambda = 0.2$. Find the probability that the volcano has two or more eruptions during the next 10 years. What is the conditional probability that the volcano has two or more eruptions during the next 10 years given that it has at least one eruption?

b) The volcano had an eruption in 1945. The intervals between the following eruptions were 7.9, 2.5, 5.4, 10.8, 10.8, 0.3, 2.6, 8.4, 13.7 and 6.8 years (the latest eruption started today.) Assume that the intervals are independent. Give an estimate of λ. (You are not required to derive the maximum likelihood estimator, but may do so if you wish.)

Problem 3

At a laboratory the connection between reaction velocity Y (in micromoles per hour) and consentration x (in micromoles per dm^3) of a catalyst is investigated. Ten measurements of recaction velocity Y_i and concentration x_i are made, $1 \leq i \leq 10$. Assume that the pairs of measurments are independent, and that Y_i has a normal distribution with expected value $\alpha + \beta x_i$ and standard deviation σ, where α, β and σ are unknown parameters.

a) Explain briefly the method of least squares for estimating α and β.

By the method of least squares the estimate of β is 1.12. The estimate of σ^2 is 2.3, and $\sum_{i=1}^{10}(x_i - \bar{x})^2 = 4.1$ (that is, 2.3/4.1 is an estimate of the variance of the estimator of β).

b) Perform a hypothesis test to investigate whether there is a connection between x and Y. Use significance level 0.05.