NTNU – Trondheim
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Department of Mathematical Sciences

# Examination paper for **ST0103 Statistics with Applications**

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**Examination date:** 7 August 2014

**Examination time (from–to):** 9:00–13:00

**Permitted examination support material:** Yellow A4 sheet with your own handwritten notes, specific basic calculator, *Tabeller og formler i statistikk* (Tapir forlag), *Matematisk formelsamling* (K. Rottmann)

**Other information:**
In the grading, each of the ten points counts equally.

You should demonstrate how you arrie at your answers (e.g. by including intermediate answers or by reffering to theory or examples from the reading list).

**Language:** English

**Number of pages:** 2

**Number pages enclosed:** 0

**Checked by:**

Date Signature

**Problem 1** A radioactive sample emits particles according to a Poisson process with intensity 1.1 (emissions per minute).

**a)** What is the probability that the sample emits exactly one particle during a time interval of half a minute? What is the probability that the sample emits at least one particle during a time interval of half a minute?

From a point of time, we measure the time before a new particle is emitted.

**b)** What is the probability that this time is greater than one minute?

**Problem 2** A proportion $p$ of an animal populations is infected by a certain parasite.

**a)** Assume (only here) that $p = 0.3$. What is the probability that two or more animals are infected by the parasite in a random sample of ten animals?

A random sample of 70 animals from the population is examined, and 29 of them are infected by the parasite.

**b)** Find an approximate 95% confidence interval for $p$.

**c)** How many animals would the sample have to consist of for the confidence interval to be guranteed to have length less than 0.2, irrespective of how many animals of the sample that are infected by the parasite?

It is known that 35% of the individuals of this species all over the country are infected by the parasite, but there are suspicions that the population from which the random sample of 70 animals are drawn, has a higher occurence of the parasite.

**d)** Test the null hypothesis $p \leq 0.35$ against the alternative hypohtesis $p > 0.35$. Use significance level 0.05.

**e)** What is the probability that the null hypothesis is rejected if we perform en experiment and a hypothesis test as described above, if $p = 0.35$? What if $p = 0.40$? Specify any approximations you make.

**Problem 3** The time of a chemical reaction (measured in milliseconds) has an exponential distribution with expected value $\mu$, that is, the probability density is given by $\frac{1}{\mu}e^{-x/\mu}$ for $x > 0$.

**a)** Assume (only here) that $\mu = 10$. What is the probability that the reaction lasts longer than 10 ms? What is the conditional probability that the reaction last longer than 20 ms given that it lasts longer than 10 ms?

**b)** Find the maximum likelihood estimator of $\mu$ based on $n$ independent observations $X_1, X_2, \ldots, X_n$ of the reaction time.

A variant of the reaction has a reaction time that is exponentially distributed with expected value $0.9\mu$. We have 5 measurements $X_1, X_2, \ldots, X_5$ of the reaction time for the reaction having expected value $\mu$ and 6 measurements $Y_1, Y_2, \ldots, Y_6$ of the reaction time for the reaction having expected value $0.9\mu$. All measurements are independent.

**c)** Which of the two estimators

$$\frac{1}{11}\left(\sum_{i=1}^{5} X_i + \frac{1}{0{,}9}\sum_{j=1}^{6} Y_j\right) \quad \text{and} \quad \frac{1}{2}\bar{X} + \frac{1}{2\cdot 0{,}9}\bar{Y}$$

for $\mu$ would you prefer?