Løsningsforslag (ST0103 2017)

1.

a)

$$E(X) = \sum_x xP(X = x) = 1\cdot0.09+2\cdot0.38+3\cdot0.25+4\cdot0.2+5\cdot0.06+6\cdot0.02 = 2.82,$$

$$E(X^2) = \sum_x x^2P(X = x) = 1\cdot0.09+4\cdot0.38+9\cdot0.25+16\cdot0.2+25\cdot0.06+36\cdot0.02 = 9.28,$$

$$Var(X) = E(X^2) - (EX)^2 = 9.28 - 7.9524 = 1.3276,$$

$$SD(X) = \sqrt{Var(X)} = \sqrt{1.3276} = 1.15,$$

$$P(X \geq 4) = P(X = 4) + P(X = 5) + P(X = 6) = 0.2 + 0.06 + 0.02 = 0.28,$$

$$P(X = 6|X \geq 4) = \frac{P(X = 6, X \geq 4)}{P(X \geq 4)} = \frac{P(X = 6)}{P(X \geq 4)} = \frac{0.02}{0.28} = 0.071.$$

b) Y er binomisk fordelt med parametre $n = 50$ og $p = 0.28$,

$$E(Y) = np = 14,\ Var(Y) = np(1 - p) = 10.08.$$

Pga sentralgrenseteoremet er Y tilnærmet normalfordelt med forventningsverdi 14 og standardavvik $\sqrt{10,8} = 3.17$. Derfor (vi bruker også heltallskorreksjon)

$$P(Y \geq 10) = P(Y \geq 10 - 0.5) = P\left(\frac{Y - 14}{3.17} \geq \frac{10 - 0.5 - 14}{3.17}\right) =$$

$$= P(Z \geq -1.42) = P(Z \leq 1.42) = 0.9222.$$

c) La A være hendelsen at reir er bygd i løvtre. Da er $P(X = 1|A) = 0.05$ og $P(X = 1|\bar{A}) = 0.15$. Vi skal finne $P(A)$. Fra total sannsynlighet formel har vi

$$P(X = 1) = P(X = 1|A)P(A) + P(X = 1|\bar{A})P(\bar{A}) =$$

$$= P(X = 1|A)P(A) + P(X = 1|\bar{A})(1 - P(A))$$

som impliserer at

$$P(A) = \frac{P(X = 1) - P(X = 1|\bar{A})}{P(X = 1|A) - P(X = 1|\bar{A})} = 0.6.$$

2.

a)

$$P\left(\mu - 2\sigma < Y < \mu + 2\sigma\right) = P\left(-2 < \frac{Y-\mu}{\sigma} < 2\right) =$$
$$= P(-2 < Z < 2) = \Phi(2) - \Phi(-2) = 2\Phi(2) - 1 = 0.9544.$$

b) The first equality implies that $\mu = 0$. Then

$$P(Y < 1) = P\left(\frac{Y}{\sigma} < \frac{1}{\sigma}\right) = P\left(Z < \frac{1}{\sigma}\right) = 0.6915$$

that implies that $1/\sigma = 0.5$ i.e. $\sigma^2 = 4$.

3.

a)

$$E\hat{\beta} = \frac{1}{n}\sum_{i=1}^{n}\frac{EY_i}{x_i} = \frac{1}{n}\sum_{i=1}^{n}\frac{\beta x_i}{x_i} = \beta.$$
$$Var\hat{\beta} = \frac{1}{n^2}\sum_{i=1}^{n}\frac{VarY_i}{x_i^2} = \frac{\sigma_0^2}{n}.$$

b) $\hat{\beta}$ has a normal distribution because it is a linear combination of independent random variables having normal distributions. So

$$\hat{\beta} \sim N(\beta, \sigma_0^2/n).$$

Therefore

$$\sqrt{n}\frac{\hat{\beta}-\beta}{\sigma_0} \sim N(0,1).$$

Then

$$P\left(-z_{\alpha/2} \le \sqrt{n}\frac{\hat{\beta}-\beta}{\sigma_0} \le z_{\alpha/2}\right) = 1 - \alpha.$$

The $100(1-\alpha)\%-$confidence interval is

$$\left[\hat{\beta} - z_{\alpha/2}\frac{\sigma_0}{\sqrt{n}}, \hat{\beta} + z_{\alpha/2}\frac{\sigma_0}{\sqrt{n}}\right]$$

4.

a) Fordelinger $N(\mu_X, \sigma^2)$ og $N(\mu_Y, \sigma^2)$. Problem:

$$H_0 : \mu_X \ge \mu_Y \quad H_1 : \mu_X < \mu_Y.$$

Testobservator

$$T = \frac{\bar{x} - \bar{y}}{s_p\sqrt{2/n}}$$

hvor

$$s_p^2 = \frac{(n-1)s_X^2 + (n-1)s_Y^2}{n+n-2} = \frac{\sum(x_i - \bar{x})^2 + \sum(y_i - \bar{y})^2}{2n-2} = 0.8591.$$

H_0 forkastes hvis $T \leq -t_{\alpha,2n-2}$. I vårt tilfelle $T = -1.1443$, $t_{\alpha,2n-2} = t_{0.05,6} = 1.943$. Derfor H_0 forkastes ikke.

b) H_0 forkastes ikke, derfor er p-verdi større enn signifikansnivå.

c) 95% konfidensintervall er

$$\left[\bar{x} - \bar{y} - t_{0.25,6} s_p \sqrt{\frac{2}{n}}, \bar{x} - \bar{y} + t_{0.25,6} s_p \sqrt{\frac{2}{n}}\right] =$$

$$= [11.175 - 11.925 - 2.447 \cdot 0.9269 \cdot 0.7071, 11.175 - 11.925 - 2.447 \cdot 0.9269 \cdot 0.7071] =$$

$$= [-2.35, 0.85].$$