NTNU
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Department of Mathematical Sciences

Examination paper for **ST0103 Statistics with applications**

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Examination time (from–to): 09:00 – 13:00

Permitted examination support material: C: Specific simple calculator. Tabeller og formler i statistikk (Tapir akademisk forlag). One yellow A4 sheet with your own handwritten notes.

Other information:

Language: English

Number of pages: 3

Number of pages enclosed: 0

Checked by:

Informasjon om trykking av eksamensoppgave
Originalen er:
1-sidig ☐ **2-sidig** ☒
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skal ha flervalgskjema ☐

Date Signature

Problem 1 A randomly chosen nest of a bird species contains X eggs. The probability distribution of X is as follows:

x	1	2	3	4	5	6
$P(X = x)$	0.09	0.38	0.25	0.20	0.06	0.02

Assume that the number of eggs in each nest is independent of each other.

a) Find the expected value and standard deviation of X. Find the probability that a randomly chosen nest has 4 or more eggs. Find the conditional probability that a randomly chosen nest has 6 eggs given it has 4 or more eggs.

b) We examine 50 randomly chosen nests. Let Y be the number of these having 4 or more eggs. What is the expected value and standard deviation of Y? Find an approximate probability that 10 or more of the 50 nests have at least 4 eggs.

c) If a nest of this bird species is built in a deciduous tree, the probability is 0.05 that it contains only one egg. If a nest is built in a coniferous tree, the probability is 0.15 that it contains only one egg. The bird species always build the nest in a deciduous tree or in a coniferous tree. What is the probability that a randomly chosen nest is built in a deciduous tree?

Problem 2

Y has the normal distribution with the expectation μ and variance σ^2.

a) Find $P(\mu - 2\sigma < Y < \mu + 2\sigma)$.

b) Let $P(Y < 0) = 0.5$ and $P(Y < 1) = 0.6915$. What are μ and σ^2?

Problem 3

In this exercise we will consider a regression model that is somewhat modified relative to what is discussed in the textbook. Assume that we have pairs of variables

$$(x_1, Y_1), (x_2, Y_2), ...(x_n, Y_n)$$

where $x_1, x_2, ..., x_n$ are positive and nonstochastic while $Y_1, Y_2, ..., Y_n$ are assumed to be independent random variables with

$$Y_i \sim \mathrm{N}(\beta x_i, \sigma_0^2 x_i^2).$$

Thus, the variance of Y_i is assumed to be proportional to x_i^2. In this exercise we will assume the value of σ_0^2 to be known while the parameter β is estimated from the available data.

a) Show that

$$\hat{\beta} = \frac{1}{n} \sum_{i=1}^{n} \frac{Y_i}{x_i}$$

is unbiased estimator of β and find the variance of $\hat{\beta}$.

b) What is the probability distribution of $\hat{\beta}$? Give reason for the answer.

Work out a $100(1-\alpha)\%$-confidence interval for β.

Problem 4

The Botanical Research Station investigated how various kinds of fertilization affect the growth of sunflowers. Two kinds of fertilization were applied. Four randomly selected sunflowers were fertilized in one way, method A, and were x_i cm taller in a week, while four other randomly selected sunflowers were fertilized in another way, method B, and were y_i cm taller. Assume that x_i and y_1 are independent observations from two normal distributions with the same variance. The growth gain of the plants were:

x_i (method A)	12.0	10.2	12.1	10.4
y_i (method B)	13.0	11.0	12.0	11.7

It is given that $\bar{x} = 11.175$, $\sum(x_i - \bar{x})^2 = 3.0875$, $\bar{y} = 11.925$, $\sum(y_i - \bar{y})^2 = 2.0675$.

a) Perform a test to investigate whether method B gives a larger expected growth gain than method A. The null hypothesis is that A gives a growth gain that is at least as large as B gives. Use significance level $\alpha = 0.05$.

b) Find a 95% confidence interval for the difference between the expected growth gains of the two fertilization methods.

Problem 5

A biologist is to estimate the number of seal pups in a population. She finds X pups. Assume that the probability p of observing a pup is known, and that X has a binomial distribution with parameters (n, p). We wish to estimate n.

a) Show that X/p is an unbiased estimator of n. Find the variance of the estimator in terms of n and p. What is the estimate if $p = 0.6$ and $X = 150$?