Løsningsforslag (ST0103 2018 august)

1.

a)

$$EX = 1 \cdot 0.09 + 2 \cdot 0.38 + 3 \cdot 0.25 + 4 \cdot 0.20 + 5 \cdot 0.06 + 6 \cdot 0.02 = 2.82$$

$$EX^2 = 1 \cdot 0.09 + 4 \cdot 0.38 + 9 \cdot 0.25 + 16 \cdot 0.20 + 25 \cdot 0.06 + 36 \cdot 0.02 = 9.28$$

$$SD(X) = \sqrt{EX^2 - (EX)^2} = \sqrt{9.28 - 2.82^2} = 1.15$$

$$P(X \geq 4) = 0.20 + 0.06 + 0.02 = 0.28$$

$$P(X = 6 | X \geq 4) = \frac{P(X = 6, X \geq 4)}{P(X \geq 4)} = \frac{P(X = 6)}{P(X \geq 4)} = \frac{0.02}{0.28} = 0.071$$

b) Y er binomisk fordelt med parametre $n = 50$ og $p = 0.28$. $EY = np = 14$, $SD(Y) = \sqrt{np(1-p)} = 3.17$. Pga av sentralgrenseteoremet

$$P(Y \geq 10) = P(Y \geq 9.5) = P\left(\frac{Y - 14}{3.17} \geq \frac{9.5 - 14}{3.17}\right) =$$

$$= P(Z \geq -1.42) = P(Z \leq 1.42) = 0.9222.$$

c) La A være hendelsen at reiret er bygd i løvtre. Da er $P(X = 1|A) = 0.05$ og $P(X = 1|\bar{A}) = 0.15$. Vi skal finne $P(A)$. Vi har

$$P(X = 1) = P(X = 1|A)P(A) + P(X = 1|\bar{A})P(\bar{A}) = 0.05P(A) + 0.15(1 - P(A)).$$

Så $P(A) = 0.6$.

2.

a)

$$P(\mu - 2\sigma < Y < \mu + 2\sigma) = P\left(-2 < \frac{Y - \mu}{\sigma} < 2\right) =$$

$$= P(-2 < Z < 2) = \Phi(2) - \Phi(-2) = 2\Phi(2) - 1 = 0.9544.$$

b) The first equality implies that $\mu = 0$. Then

$$P(Y < 1) = P\left(\frac{Y}{\sigma} < \frac{1}{\sigma}\right) = P\left(Z < \frac{1}{\sigma}\right) = 0.6915$$

that implies that $1/\sigma = 0.5$ i.e. $\sigma^2 = 4$.

3.

a)

$$E\hat{\beta} = \frac{1}{n}\sum_{i=1}^{n}\frac{EY_i}{x_i} = \frac{1}{n}\sum_{i=1}^{n}\frac{\beta x_i}{x_i} = \beta.$$

$$Var\hat{\beta} = \frac{1}{n^2}\sum_{i=1}^{n}\frac{VarY_i}{x_i^2} = \frac{\sigma_0^2}{n}.$$

b) $\hat{\beta}$ has a normal distribution because it is a linear combination of independent random variables having normal distributions. So

$$\hat{\beta} \sim N(\beta, \sigma_0^2/n).$$

Therefore

$$\sqrt{n}\frac{\hat{\beta}-\beta}{\sigma_0} \sim N(0,1).$$

Then

$$P\left(-z_{\alpha/2} \le \sqrt{n}\frac{\hat{\beta}-\beta}{\sigma_0} \le z_{\alpha/2}\right) = 1-\alpha.$$

The $(1-\alpha)-$confidence interval is

$$\left[\hat{\beta} - z_{\alpha/2}\frac{\sigma_0}{\sqrt{n}}, \hat{\beta} + z_{\alpha/2}\frac{\sigma_0}{\sqrt{n}}\right]$$

4.

a)

$$H_0 : \mu_X \ge \mu_y, \quad H_1 : \mu_X < \mu_y.$$

Bruker uparet t-test.

$$s_p^2 = \frac{3.0875 + 2.0675}{4+4-2} = 0.8592,$$

$$t = \frac{11.175 - 11.925}{\sqrt{0.8592}\sqrt{1/4+1/4}} = -1.1443.$$

Forkaster H_0 hvis t er liten. $-t_{0.05} = -1.943$. Forkaster ikke H_0.

b) $t_{0.025} = 2.447$. 95%-konfidensintervall har grenser

$$11.175 - 11.925 \pm 2.447 \cdot \sqrt{0.8592}\sqrt{1/4+1/4},$$

dvs. intervallet er $[-2.35, 0.85]$.

5.

a) Forventningverdien til estimatoren er

$$E\frac{X}{p} = \frac{1}{p}EX = \frac{1}{p}np = n,$$

så estimatoren er forventningsrett. Variansen er

$$\mathrm{Var}\frac{X}{p} = \frac{1}{p^2}\mathrm{Var}X = \frac{1}{p^2}np(1-p) = \frac{n(1-p)}{p}.$$

Estimatet blir $150/0.6 = 250$.