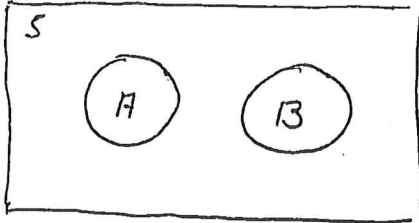


ST 1101

SANNSYNSREKNING	FORDELINGAR	INFERENS
SANNSYNN	TÅLFELDIGE VARIABLE	SENTRALGRENSE -
ADDISJONSSETNINGAR	FORVENTNING	TEOREMET
MULTIPLIKASJONS-	VARIANS	ESTIMERING
SETNINGAR	STANDARDAVVIK	KONFIDENSINTERVALL
BETINGA SANNSYNN	KOVARIANS	HYPOTHESETESTING
TOTAL SANNSYNN	UNIFORM	
BAYES FORMEL	BINOMISK	
KOMBINATORIKK	GEOMETRISK	
	NEGATIVT BINOMISK	
	HYPERGEOMETRISK	
	POISSON	
	NORMAL	
	EKSPONENTIÅL	
	GAMMA	

SANNSYN

STOKASTISKE FORSØK

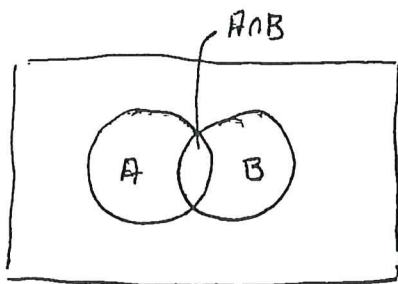


$$P(S) = 1$$

$$P(A) \geq 0$$

$$P(A \cup B) = P(A) + P(B), \quad A \cap B = \emptyset$$

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i), \quad A_i \cap A_j = \emptyset, \quad \forall i, j, i \neq j$$



$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

BETINGTA SANNSYN:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

UAVHÆNGIGHED:

$$P(A|B) = P(A), \quad P(A|B^c) = P(A)$$

$$P(A \cap B) = P(A) \cdot P(B)$$

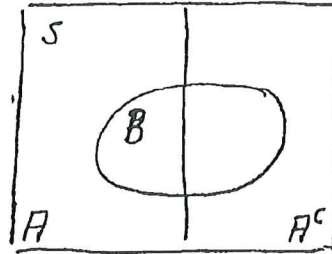
$$P(A_1 \cap \dots \cap A_m) = \prod_{i=1}^m P(A_i)$$

$$P(A_{i_1} \cap A_{i_2} \cap \dots \cap A_{i_j}) = P(A_{i_1}) \cdot P(A_{i_2}) \cdot \dots \cdot P(A_{i_j})$$

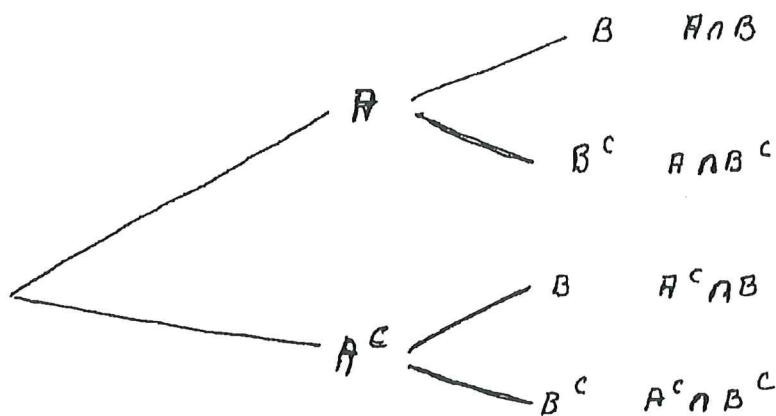
$\forall j$ og $i_1, \dots, i_j$

BAYES REGEL

TOTAL SAMNSYN



$$P(B) = P(A \cap B) + P(A^c \cap B)$$



$$P(A|B) = \frac{P(A \cap B)}{P(A \cap B) + P(A^c \cap B)} \quad \text{Bayes regel}$$

$$= \frac{P(B|A) \cdot P(A)}{P(B|A) \cdot P(A) + P(B|A^c) \cdot P(A^c)}$$

$$P(B|A) \cdot P(A) + P(B|A^c) \cdot P(A^c)$$

KOMBINATORIKK

UNIFORME SANNSYNSMODELLER $P(A) = \frac{g}{m}$

MULTIPLIKASJONSREGELEN: $m_1 \cdot m_2 \cdot \dots \cdot m_k$.

TALET PÅ ORDNA UTVAL $\frac{m!}{(m-s)!} = m(m-1) \cdot \dots \cdot (m-s+1)$

TALET PÅ IKKJE ORDNA UTVAL $\frac{m!}{(m-s)! \cdot s!} = \binom{m}{s}$

FÖRDELINGSFUNKTIONAR

Diskret

$$F_X(x) = P(X \leq x) = \sum_{u \leq x} P(X=u)$$

$x = k$ heltal

$$F_X(k) = P(X \leq k) = \sum_{i=k}^{\infty} P(X=i)$$

$$P(X > k) = P(X \geq k+1) = \sum_{i=k+1}^{\infty} P(X=i)$$

$$P(X=k) = P(X \leq k) - P(X \leq k-1)$$

Kontinuerlig

$$F_Y(y) = P(Y \leq y) = \int_{-\infty}^y f_Y(y) dy$$

$$f_Y(y) = \frac{d}{dy} F_Y(y)$$

$$P(Y=y) = 0$$

FLEIRDIMENSIONALE VERTEILUNGSFUNKTIONEN

Diskret

$$F_{X,Y}(x,y) = P(X \leq x \cap Y \leq y) = \sum_{v \leq y} \sum_{u \leq x} P(X=u \cap Y=v)$$

$$P(X=x) = \sum_{\text{alle } y} P(X=x \cap Y=y)$$

Kontinuierlich

$$F_{X,Y}(x,y) = \int_{-\infty}^y \int_{-\infty}^x f_{X,Y}(u,v) du dv$$

$$f_Y(y) = \int_{-\infty}^{\infty} f_{X,Y}(x,y) dx$$

Unabh.

$$P(X_1=x_1 \cap X_2=x_2 \cap \dots \cap X_m=x_m) = \prod_{i=1}^m P(X_i=x_i)$$

$$f_{X_1, \dots, X_m}(y_1, \dots, y_m) = \prod_{i=1}^m f_{X_i}(y_i)$$

VERWENTUNG EINER FUNKTION

$$E[g(X)] = \begin{cases} \sum_{x_i} g(x_i) P(X=x_i), & X \text{ diskret} \\ \int_{-\infty}^{\infty} g(x) f(x) dx, & X \text{ kontinuierlich} \end{cases}$$

$$E[g(X,Y)] = \begin{cases} \sum_y \sum_x g(x,y) P(X=x \cap Y=y), & \text{Diskret} \\ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x,y) f_{X,Y}(x,y) dx dy, & \text{kontinuierlich} \end{cases}$$

FORVENTNING OG VARIANS

$$E[X] = \begin{cases} \int_{-\infty}^{\infty} x f(x) dx, & X \text{ kontinuert} \\ \sum_i x_i P(X=x_i), & X \text{ diskret} \end{cases}$$

$$\text{Var}[X] = \begin{cases} \int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx, & X \text{ kontinuert} \\ \sum_i (x_i-\mu)^2 P(X=x_i), & X \text{ diskret} \end{cases}$$

$$\text{Cov}(X, Y) = E[(X-\mu_X)(Y-\mu_Y)]$$

$$\rho(X, Y) = \frac{\text{Cov}(X, Y)}{\sigma_X \sigma_Y}$$

$$E[\sum a_i X_i] = \sum a_i E[X_i]$$

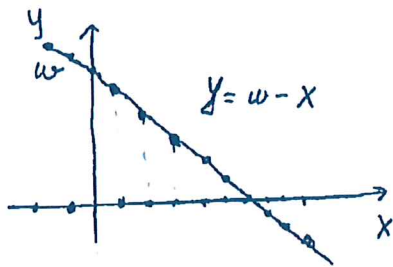
$$\text{Var}[\sum a_i X_i] = \sum a_i^2 \text{Var}[X_i] + 2 \sum_{i < j} a_i a_j \text{Cov}(X_i, X_j)$$

KONVOLUSIONSETZUNG

$$W = X + Y = g(X, Y)$$

X, Y unabh.

$$P(W=w) = \sum_{\text{alle } x} P(X=x \cap Y=w-x)$$



Diskret

Kontinuierlich

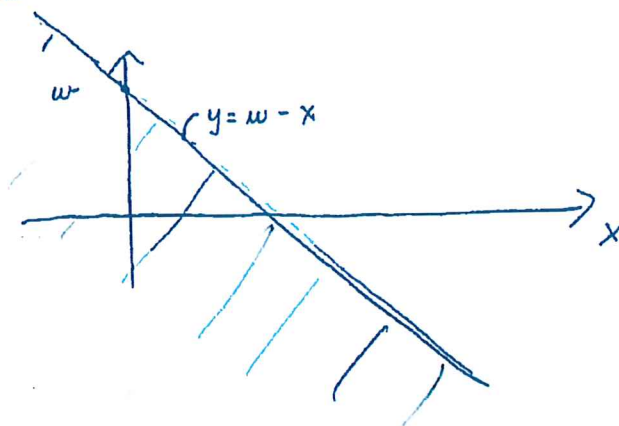
X, Y unabh.

$$W = X + Y$$

$$F_W(w) = P(W \leq w) = P(X + Y \leq w) = P(Y \leq w - X)$$

$$= \int_{-\infty}^{\infty} \left(\int_{-\infty}^{w-x} f_X(x) f_Y(y) dy \right) dx = \int_{-\infty}^{\infty} f_X(x) F_Y(w-x) dx$$

$$\frac{dF_W(w)}{dw} = \int_{-\infty}^{\infty} f_X(x) f_Y(w-x) dx$$



MAX OG MIN

$X_1, \dots, X_m$ uafhængige

$$P(\max(X_1, \dots, X_m) \leq x) = P(X_1 \leq x \cap X_2 \leq x \cap \dots \cap X_m \leq x)$$

$$= \prod_{i=1}^m F_{X_i}(x)$$

$$P(\min(X_1, \dots, X_m) \leq x) = 1 - P(X_1 > x \cap X_2 > x \cap \dots \cap X_m > x)$$

$$= 1 - \prod_{i=1}^m P(X_i > x) = 1 - \prod_{i=1}^m (1 - F_{X_i}(x))$$

BETINGA FORDELINGER

$$P(Y=y | X=x) = \frac{P(\cancel{Y=y} \cap X=x)}{P(X=x)}, \quad \text{Diskret}$$

$$f_{Y|X=x} = \frac{f_{X,Y}(x,y)}{f_X(x)}, \quad \text{kontinuerlig}$$

CENTRAL GRENSE TEOREMET

$X_1, \dots, X_m$ hlf. utval $\left\{ \begin{array}{l} \text{uavh.} \\ \text{identiske fordelte} \end{array} \right.$

$$E[X_i] = \mu, \quad \text{Var}[X_i] = \sigma^2$$

$$E[\bar{X}] = \mu$$

$$\text{Var}[\bar{X}] = \frac{\sigma^2}{m}$$

$$\frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{m}}} \xrightarrow{m \rightarrow \infty} N(0, 1)$$

eller: $\bar{X} \approx N\left(\mu, \frac{\sigma^2}{m}\right)$

eller: $\sum_{i=1}^m X_i \approx N(m\mu, m\sigma^2)$

DISKRETE FORDELINGAR

Binomisk (n, p)

X = tal på gange A sker i n forsøk

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$$E[X] = np, \quad \text{Var}[X] = np(1-p)$$

Geometrisk (p)

X = tal på forsøk til A skjer 1. gang

$$P(X=k) = (1-p)^{k-1} p$$

$$E[X] = \frac{1}{p}, \quad \text{Var}[X] = \frac{1-p}{p^2}$$

Negativ Binomisk (n, p)

X = tal på forsøk til A skjer n -te gang.

$$P(X=k) = \binom{k-1}{n-1} p^n (1-p)^{k-n}$$

$$E[X] = \frac{n}{p}, \quad \text{Var}[X] = \frac{n(1-p)}{p^2}$$

Hypergeometrisk

X = tal av type A i et utval av størrelse m fra N der n stykker er av type A

$$P(X=k) = \frac{\binom{n}{k} \binom{N-n}{m-k}}{\binom{N}{m}}$$

$$E[X] = m \frac{n}{N}, \quad \text{Var}[X] = \frac{N-m}{N-1} m \frac{n}{N} \left(1 - \frac{n}{N}\right)$$

TILNÆRMINGAR/RELASJONAR

pluton, $mp = \lambda$

$$P(X=k) = \frac{(mp)^k e^{-mp}}{k!}$$

$$\left. \begin{array}{l} mp \geq 5 \\ m(1-p) \geq 5 \end{array} \right\} \Rightarrow X \approx N(mp, mp(1-p))$$

$X_i, i=1, 2, \dots, n$ uavh og

geometrisk (p)

$\sum_{i=1}^n X_i \sim \text{Negativ Binomisk}(n, p)$

$$\approx N\left(\frac{n}{p}, \frac{n(1-p)}{p^2}\right)$$

$$N \gg m \Rightarrow$$

$$P(X=k) \approx \binom{m}{k} \left(\frac{n}{N}\right)^k \left(1 - \frac{n}{N}\right)^{m-k}$$

$\approx$

Poisson (λt)

X = tal på po^0 ganger A skjer
over tid (po^0 flate, volum o.l.)

$$P(X=k) = \frac{e^{-\lambda t} (\lambda t)^k}{k!}$$

$$E[X] = \lambda t, \quad \text{Var}[X] = \lambda t$$

KONTINUERLEGE FORDELINGAR

Normal (μ, σ^2)

$$f(y; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{1}{2\sigma^2}(y-\mu)^2}$$

$-\infty < y < \infty$

$$E[Y] = \mu, \quad \text{Var}[Y] = \sigma^2$$

eksponential (λ)

$$f(y; \lambda) = \begin{cases} \lambda e^{-\lambda y}, & y > 0 \\ 0, & \text{ellers.} \end{cases}$$

$$E[Y] = \frac{1}{\lambda}, \quad \text{Var}[Y] = \frac{1}{\lambda^2}$$

Gammafordeling (λ, n)

$$f(y, \lambda, n) = \frac{\lambda^n}{\Gamma(n)} y^{n-1} e^{-\lambda y}, \quad y > 0$$

$$E[Y] = \frac{n}{\lambda}, \quad \text{Var}[Y] = \frac{n}{\lambda^2}$$

TILNÆRMINGAR/RELASJONAR

$$\lambda t \geq 18$$

$$X \approx N(\lambda t, \lambda t)$$

Verthid til 1. hendring (mellom 2 hendringer) i Poisson-prosessen.

n heeltal: Ventetid til n -te hendring (mellom $n-1$ hendring og den n -te hendring) i Poissonprosessen.

$Y_1, Y_2, \dots, Y_n$ eksponentialfordelte (λ)
og uavh. $\Rightarrow \sum_{i=1}^n Y_i \sim \text{gamma}(\lambda, n)$
 $\approx N\left(\frac{n}{\lambda}, \frac{n}{\lambda^2}\right)$

MOMENT GENERERANDE FUNKTIONER

$$M_X(t) = E[e^{t^T X}]$$

$$M_{aX+b}(t) = e^{bt^T} M_X(at)$$

TO RESULTAT

$$E[X^n] = \left. \frac{d^n}{dt^n} M_X(t) \right|_{t=0}$$

$$M_{\sum_{i=1}^m X_i}(t) = \prod_{i=1}^m M_{X_i}(t)$$

uavh

$X_1, X_2, \dots, X_m$ uavh

$$X_i \sim \text{Poisson}(\lambda_i) \Rightarrow \sum_{i=1}^m X_i \sim \text{Poisson}(\sum_{i=1}^m \lambda_i)$$

$$X_i \sim B(m_i, p) \Rightarrow \sum_{i=1}^m X_i \sim B(\sum_{i=1}^m m_i, p)$$

$$X_i \sim N(\mu_i, \sigma_i^2) \Rightarrow \sum_{i=1}^m X_i \sim N(\sum_{i=1}^m \mu_i, \sum_{i=1}^m \sigma_i^2)$$

$$X_i \sim \text{geometrisk}(p) \Rightarrow \sum_{i=1}^m X_i \sim \text{Negativ binomisk}(m, p)$$

$$X_i \sim \text{negativ binomisk}(n_i, p) \Rightarrow \sum_{i=1}^m X_i \sim \text{Negativ binomisk}(\sum_{i=1}^m n_i, p)$$

$$X_i \sim \text{eksponentialfordelt}(\lambda) \Rightarrow \sum_{i=1}^m X_i \sim \text{gammafordelt}(m, \lambda)$$

$$X_i \sim \text{gammafordelt}(n_i, \lambda) \Rightarrow \sum_{i=1}^m X_i \sim \text{gammafordelt}(\sum_{i=1}^m n_i, \lambda)$$

INFERENCE

$X_1, \dots, X_m$ i.i.d.

$$L(\theta) = \begin{cases} f_{X_1, \dots, X_m}(x_1, \dots, x_m; \theta) = \prod_{i=1}^m f_{X_i}(x_i; \theta) \\ P(X_1=x_1, X_2=x_2, \dots, X_m=x_m, \theta) = \prod_{i=1}^m P(X_i=x_i; \theta) \end{cases}$$

SSE: θ_c maximizes $L(\theta)$, wdt. $\ln L(\theta)$

$\theta_c = \theta(x_1, \dots, x_m)$ estimate.

$\hat{\theta} = \hat{\theta}(X_1, \dots, X_m)$ estimator.

Moment estimators: $E[Y^k] = \frac{1}{m} \sum_{i=1}^m y_i^k, k=1, 2, \dots$

Konfidenzintervall

$$P\left(-z_{\frac{\alpha}{2}} \leq \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{m}}} \leq z_{\frac{\alpha}{2}}\right) = 1 - \alpha$$

$$\Leftrightarrow P\left(\bar{X} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{m}}, \bar{X} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{m}}\right) = 1 - \alpha.$$

Hypotesetesting i normalfordeling

$$H_0: \mu \begin{cases} \leq \\ = \\ \geq \end{cases} \mu_0 \quad H_1: \mu \begin{cases} > \\ \neq \\ < \end{cases} \mu_0$$

$$\text{Forkast } H_0 \text{ om } \left\{ \begin{array}{l} Z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \geq z_\alpha \\ |Z| = \left| \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \right| \geq z_{\frac{\alpha}{2}} \\ Z = \frac{\bar{X} - \mu_0}{\frac{\sigma}{\sqrt{n}}} \leq -z_\alpha \end{array} \right\}$$

P-verdien er sannsynet for å få noko som er minst like ekstremt som det vi har vi har observert gitt H_0 .

P-verdien er det minste signifikansnivået som fører til forkastning av H_0 .

$$P(Z \geq z_{\text{obs}} | H_0), \quad 2P(Z \geq |z_{\text{obs}}| | H_0), \quad P(Z \leq z_{\text{obs}} | H_0)$$

Type I og Type II feil.

Type I feil: $P(\text{forkaste } H_0 \mid H_0 \text{ er rett}) = \alpha$

Type 2 feil: $P(\text{ikke å forkaste } H_0 \mid H_0 \text{ er gal}) = \beta$

Styrkefunksjonen $P(\text{å forkaste } H_0 \mid H_0 \text{ er gal}) = 1 - \beta$

Normalfordeling: $H_0: \mu = \mu_0$ $H_1: \mu > \mu_0$

Styrkefunksjonen $1 - \Phi\left(z_\alpha - \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}\right)$

Normalfordeling $H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$

Styrkefunksjonen $1 - \left[\Phi\left(z_{\frac{\alpha}{2}} - \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}\right) - \Phi\left(-z_{\frac{\alpha}{2}} - \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}\right) \right]$

