Questions

3.4.1. Suppose $f_Y(y) = 4y^3, 0 \le y \le 1$. Find $P\left(0 \le Y \le \frac{1}{2}\right)$.

3.4.2. For the random variable Y with pdf $f_Y(y) = \frac{2}{3} + \frac{2}{3}y, 0 \le y \le 1$, find $P\left(\frac{3}{4} \le Y \le 1\right)$.

3.4.3. Let $f_Y(y) = \frac{3}{2}y^2, -1 \le y \le 1$. Find $P\left(|Y - \frac{1}{2}| < \frac{1}{4}\right)$. Draw a graph of $f_Y(y)$ and show the area representing the desired probability.

3.4.4. For persons infected with a certain form of malaria, the length of time spent in remission is described by the continuous pdf $f_Y(y) = \frac{1}{9}y^2, 0 \le y \le 3$, where Y is measured in years. What is the probability that a malaria patient's remission lasts longer than one year?

3.4.5. The length of time, Y, that a customer spends in line at a bank teller's window before being served is described by the exponential pdf $f_Y(y) = 0.2e^{-0.2y}, y \ge 0$.

(a) What is the probability that a customer will wait more than ten minutes?

(b) Suppose the customer will leave if the wait is more than ten minutes. Assume that the customer goes to the bank twice next month. Let the random variable X be the number of times the customer leaves without being served. Calculate $p_X(1)$.

3.4.6. Let n be a positive integer. Show that $f_Y(y) = (n+2)(n+1)y^n(1-y), 0 \le y \le 1$, is a pdf.

3.4.7. Find the cdf for the random variable Y given in Question 3.4.1. Calculate $P\left(0 \le Y \le \frac{1}{2}\right)$ using $F_Y(y)$.

3.4.8. If Y is an exponential random variable, $f_Y(y) = \lambda e^{-\lambda y}, y \ge 0$, find $F_Y(y)$.

3.4.9. If the pdf for Y is

$$f_Y(y) = \begin{cases} 0, & |y| > 1 \\ 1 - |y|, & |y| \le 1 \end{cases}$$

find and graph $F_Y(y)$.

3.4.10. A continuous random variable Y has a cdf given by

$$F_Y(y) = \begin{cases} 0 & y < 0 \\ y^2 & 0 \le y < 1 \\ 1 & y \ge 1 \end{cases}$$

Find $P\left(\frac{1}{2} < Y \le \frac{3}{4}\right)$ two ways—first, by using the cdf and second, by using the pdf.

3.4.11. A random variable Y has cdf

$$F_Y(y) = \begin{cases} 0 & y < 1 \\ \ln y & 1 \le y \le e \\ 1 & e < y \end{cases}$$

Find

(a) $P(Y < 2)$
(b) $P\left(2 < Y \le 2\frac{1}{2}\right)$
(c) $P\left(2 < Y < 2\frac{1}{2}\right)$
(d) $f_Y(y)$

3.4.12. The cdf for a random variable Y is defined by $F_Y(y) = 0$ for $y < 0$; $F_Y(y) = 4y^3 - 3y^4$ for $0 \le y \le 1$; and $F_Y(y) = 1$ for $y > 1$. Find $P\left(\frac{1}{4} \le Y \le \frac{3}{4}\right)$ by integrating $f_Y(y)$.

3.4.13. Suppose $F_Y(y) = \frac{1}{12}(y^2 + y^3), 0 \le y \le 2$. Find $f_Y(y)$.

3.4.14. In a certain country, the distribution of a family's disposable income, Y, is described by the pdf $f_Y(y) = ye^{-y}, y \ge 0$. Find $F_Y(y)$.

3.4.15. The logistic curve $F(y) = \frac{1}{1+e^{-y}}, -\infty < y < \infty$, can represent a cdf since it is increasing, $\lim_{y \to -\infty} \frac{1}{1+e^{-y}} = 0$, and $\lim_{y \to +\infty} \frac{1}{1+e^{-y}} = 1$. Verify these three assertions and also find the associated pdf.

3.4.16. Let Y be the random variable described in Question 3.4.1. Define $W = 2Y$. Find $f_W(w)$. For which values of w is $f_W(w) \ne 0$?

3.4.17. Suppose that $f_Y(y)$ is a continuous and symmetric pdf, where *symmetry* is the property that $f_Y(y) = f_Y(-y)$ for all y. Show that $P(-a \le Y \le a) = 2F_Y(a) - 1$.

3.4.18. Let Y be a random variable denoting the age at which a piece of equipment fails. In reliability theory, the probability that an item fails at time y given that it has survived until time y is called the *hazard rate*, $h(y)$. In terms of the pdf and cdf,

$$h(y) = \frac{f_Y(y)}{1 - F_Y(y)}$$

Find $h(y)$ if Y has an exponential pdf (see Question 3.4.8).

Questions

3.5.1. Recall the game of Keno described in Question 3.2.26. The following are all the payoffs on a $1 wager where the player has bet on ten numbers. Calculate $E(X)$, where the random variable X denotes the amount of money won.

Number of Correct Guesses	Payoff	Probability
< 5	−$ 1	.935
5	2	.0514
6	18	.0115
7	180	.0016
8	1,300	1.35×10^{-4}
9	2,600	6.12×10^{-6}
10	10,000	1.12×10^{-7}

3.5.2. The roulette wheels in Monte Carlo typically have a 0 but not a 00. What is the expected value of betting on red in this case? If a trip to Monte Carlo costs $3000, how much would a player have to bet to justify gambling there rather than Las Vegas?

3.5.3. The pdf describing the daily profit, X, earned by Acme Industries was derived in Example 3.3.7. Find the company's *average* daily profit.

3.5.4. In the game of redball, two drawings are made without replacement from a bowl that has four white ping-pong balls and two red ping-pong balls. The amount won is determined by how many of the red balls are selected. For a $5 bet, a player can opt to be paid under either Rule A or Rule B, as shown. If you were playing the game, which would you choose? Why?

A		*B*	
No. of Red Balls Drawn	Payoff	No. of Red Balls Drawn	Payoff
0	0	0	0
1	$2	1	$1
2	$10	2	$20

3.5.5. Suppose a life insurance company sells a $50,000, five-year term policy to a twenty-five-year-old woman. At the beginning of each year the woman is alive, the company collects a premium of $\$P$. The probability that the woman dies and the company pays the $50,000 is given in the table below. So, for example, in Year 3, the company loses $\$50{,}000 - \$P$ with probability 0.00054 and gains $\$P$ with probability $1 - 0.00054 = 0.99946$. If the company expects to make $1000 on this policy, what should P be?

Year	Probability of Payoff
1	0.00051
2	0.00052
3	0.00054
4	0.00056
5	0.00059

3.5.6. A manufacturer has one hundred memory chips in stock, 4% of which are likely to be defective (based on past experience). A random sample of twenty chips is selected and shipped to a factory that assembles laptops. Let X denote the number of computers that receive faulty memory chips. Find $E(X)$.

3.5.7. Records show that 642 new students have just entered a certain Florida school district. Of those 642, a total of 125 are not adequately vaccinated. The district's physician has scheduled a day for students to receive whatever shots they might need. On any given day, though, 12% of the district's students are likely to be absent. How many new students, then, can be expected to remain inadequately vaccinated?

3.5.8. Calculate $E(Y)$ for the following pdfs:

(a) $f_Y(y) = 3(1-y)^2,\ 0 \le y \le 1$

(b) $f_Y(y) = 4ye^{-2y},\ y \ge 0$

(c) $f_Y(y) = \begin{cases} \frac{3}{4}, & 0 \le y \le 1 \\ \frac{1}{4}, & 2 \le y \le 3 \\ 0, & \text{elsewhere} \end{cases}$

(d) $f_Y(y) = \sin y,\quad 0 \le y \le \frac{\pi}{2}$

3.5.9. Recall Question 3.4.4, where the length of time Y (in years) that a malaria patient spends in remission has pdf $f_Y(y) = \frac{1}{9}y^2, 0 \le y \le 3$. What is the average length of time that such a patient spends in remission?

3.5.10. Let the random variable Y have the uniform distribution over $[a, b]$; that is, $f_Y(y) = \frac{1}{b-a}$ for $a \le y \le b$. Find $E(Y)$ using Definition 3.5.1. Also, deduce the value of $E(Y)$, knowing that the expected value is the center of gravity of $f_Y(y)$.

3.5.11. Show that the expected value associated with the exponential distribution, $f_Y(y) = \lambda e^{-\lambda y}$, $y > 0$, is $1/\lambda$, where λ is a positive constant.

3.5.12. Show that

$$f_Y(y) = \frac{1}{y^2}, \quad y \ge 1$$

is a valid pdf but that Y does not have a finite expected value.

3.5.13. Based on recent experience, ten-year-old passenger cars going through a motor vehicle inspection station have an 80% chance of passing the emissions test. Suppose that two hundred such cars will be checked out next week. Write two formulas that show the number of cars that are expected to pass.

3.5.14. Suppose that fifteen observations are chosen at random from the pdf $f_Y(y) = 3y^2, 0 \le y \le 1$. Let X denote the number that lie in the interval $\left(\frac{1}{2}, 1\right)$. Find $E(X)$.

3.5.15. A city has 74,806 registered automobiles. Each is required to display a bumper decal showing that the owner paid an annual wheel tax of \$50. By law, new decals need to be purchased during the month of the owner's birthday. How much wheel tax revenue can the city expect to receive in November?

3.5.16. Regulators have found that twenty-three of the sixty-eight investment companies that filed for bankruptcy in the past five years failed because of fraud, not for reasons related to the economy. Suppose that nine additional firms will be added to the bankruptcy rolls during the next quarter. How many of those failures are likely to be attributed to fraud?

3.5.17. An urn contains four chips numbered 1 through 4. Two are drawn without replacement. Let the random variable X denote the larger of the two. Find $E(X)$.

3.5.18. A fair coin is tossed three times. Let the random variable X denote the total number of heads that appear times the number of heads that appear on the first and third tosses. Find $E(X)$.

3.5.19. How much would you have to ante to make the St. Petersburg game "fair" (recall Example 3.5.5) if the most you could win was \$1000? That is, the payoffs are $\$2^k$ for $1 \le k \le 9$, and \$1000 for $k \ge 10$.

3.5.20. For the St. Petersburg problem (Example 3.5.5), find the expected payoff if

(a) the amounts won are c^k instead of 2^k, where $0 < c < 2$.
(b) the amounts won are $\log 2^k$. [This was a modification suggested by D. Bernoulli (a nephew of James Bernoulli) to take into account the decreasing marginal utility of money—the more you have, the less useful a bit more is.]

3.5.21. A fair die is rolled three times. Let X denote the number of different faces showing, $X = 1, 2, 3$. Find $E(X)$.

3.5.22. Two distinct integers are chosen at random from the first five positive integers. Compute the expected value of the absolute value of the difference of the two numbers.

3.5.23. Suppose that two evenly matched teams are playing in the World Series. On the average, how many games will be played? (The winner is the first team to get four victories.) Assume that each game is an independent event.

3.5.24. An urn contains one white chip and one black chip. A chip is drawn at random. If it is white, the "game" is over; if it is black, that chip and another black one are put into the urn. Then another chip is drawn at random from the "new" urn and the same rules for ending or continuing the game are followed (i.e., if the chip is white, the game is over; if the chip is black, it is placed back in the urn, together with another chip of the same color). The drawings continue until a white chip is selected. Show that the expected number of drawings necessary to get a white chip is not finite.

3.5.25. A random sample of size n is drawn without replacement from an urn containing r red chips and w white chips. Define the random variable X to be the number of red chips in the sample. Use the summation technique described in Theorem 3.5.1 to prove that $E(X) = rn/(r + w)$.

3.5.26. Given that X is a nonnegative, integer-valued random variable, show that

$$E(X) = \sum_{k=1}^{\infty} P(X \ge k)$$

3.5.27. Find the median for each of the following pdfs:

(a) $f_Y(y) = (\theta + 1)y^\theta,\ 0 \le y \le 1$, where $\theta > 0$
(b) $f_Y(y) = y + \frac{1}{2},\ 0 \le y \le 1$

Questions

3.5.28. Suppose X is a binomial random variable with $n = 10$ and $p = \frac{2}{5}$. What is the expected value of $3X - 4$?

3.5.29. A typical day's production of a certain electronic component is twelve. The probability that one of these components needs rework is 0.11. Each component needing rework costs \$100. What is the average daily cost for defective components?

3.5.30. Let Y have probability density function

$$f_Y(y) = 2(1 - y),\ 0 \le y \le 1$$

Suppose that $W = Y^2$, in which case

$$f_W(w) = \frac{1}{\sqrt{w}} - 1,\ 0 \le w \le 1$$

Find $E(W)$ in two different ways.

3.5.31. A tool and die company makes castings for steel stress-monitoring gauges. Their annual profit, Q, in hundreds of thousands of dollars, can be expressed as a function of product demand, y:

$$Q(y) = 2(1 - e^{-2y})$$

Suppose that the demand (in thousands) for their castings follows an exponential pdf, $f_Y(y) = 6e^{-6y}$, $y > 0$. Find the company's expected profit.

3.5.32. A box is to be constructed so that its height is five inches and its base is Y inches by Y inches, where Y is a random variable described by the pdf, $f_Y(y) = 6y(1 - y)$, $0 < y < 1$. Find the expected volume of the box.

3.5.33. Grades on the last Economics 301 exam were not very good. Graphed, their distribution had a shape similar to the pdf

$$f_Y(y) = \frac{1}{5000}(100 - y), \quad 0 \le y \le 100$$

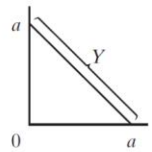
As a way of "curving" the results, the professor announces that he will replace each person's grade, Y, with a new grade, $g(Y)$, where $g(Y) = 10\sqrt{Y}$. Will the professor's strategy be successful in raising the class average above 60?

3.5.34. If Y has probability density function

$$f_Y(y) = 2y,\ 0 \le y \le 1$$

then $E(Y) = \frac{2}{3}$. Define the random variable W to be the squared deviation of Y from its mean, that is, $W = \left(Y - \frac{2}{3}\right)^2$. Find $E(W)$.

3.5.35. The hypotenuse, Y, of the isosceles right triangle shown is a random variable having a uniform pdf over the interval [6, 10]. Calculate the expected value of the triangle's area. Do not leave the answer as a function of a.



3.5.36. An urn contains n chips numbered 1 through n. Assume that the probability of choosing chip i is equal to ki, $i = 1, 2, \ldots, n$. If one chip is drawn, calculate $E\left(\frac{1}{X}\right)$, where the random variable X denotes the number showing on the chip selected. [*Hint*: Recall that the sum of the first n integers is $n(n + 1)/2$.]