ST0101 Brukerkurs i Sannsynlighetsregning

Løsningsforslag

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Oppg. 1 Fugleredet

a) $P(X \geq 2) = \sum_{x=2}^{4} P(X=x) = 0.3+0.3+0.1 = \underline{\underline{0.7}}$

$$\mu_x = E(X) = \sum_{x=0}^{4} x\, P(X=x) = 0\cdot 0.1 + 1\cdot 0.2 + 2\cdot 0.3 + 3\cdot 0.3 + 4\cdot 0.1$$

$$= 0 + 0.2 + 0.6 + 0.9 + 0.4$$

$$= \underline{\underline{2.1}}$$

$$\sigma_x^2 = Var(X) = \sum_{x=0}^{4} (x-\mu_x)^2 P(X=x) = \sum_{x=0}^{4} x^2 P(X=x) - \mu_x^2$$

$$= 0^2\cdot 0.1 + 1^2\, 0.2 + 2^2\, 0.3 + 3^2\cdot 0.3 + 4^2\, 0.1 - 2.1^2$$

$$= 0 + 0.2 + 1.2 + 2.7 + 1.6 - 4.41$$

$$= \underline{\underline{1.29}}$$

b) $P(Y=0) = \sum_{x=0}^{4} P(Y=0 \mid X=x)\, P(X=x)$

$$= 1.0\cdot 0.1 + 0.6\cdot 0.2 + 0.2\cdot 0.3 + 0.1\cdot 0.3 + 0\cdot$$

$$= 0.1 + 0.12 + 0.06 + 0.03 + 0$$

$$= \underline{\underline{0.31}}$$

$$E(Y) = E_X(E_Y(Y \mid X)) = \sum_{x=0}^{4} E_Y(Y \mid X=x)\, P(X=x)$$

$$= 0\cdot 0.1 + 0.4\cdot 0.2 + 1.0\cdot 0.3 + 1.7\cdot 0.3 + 2.5\cdot 0.1$$

$$= 0 + 0.08 + 0.3 + 0.51 + 0.25$$

$$= \underline{\underline{1.14}}$$

$$Var(Y) = E_X(Var_Y(Y|X)) + Var_X(E_Y(Y|X))$$

$$= E_X(E_Y(Y^2|X)) - [E_Y(Y)]^2$$

$$= \sum_{x=0}^{4} E_Y(Y^2|X=x)P(X=x) - [E_Y(Y)]^2$$

$$= 0\cdot 0.1 + 0.4\cdot 0.2 + 1.4\cdot 0.3 + 3.5\cdot 0.3 + 6.9\cdot 0.1 - (1.14)^2$$

$$= 2.24 - (1.14)^2 = \underline{\underline{\qquad}}$$

alt. regn ut $P(Y=y); y=0,1,..,4$ og $Var(Y)$ fra det!!

c) $$P(X=x|Y=y) = \frac{P(X=x \cap Y=y)}{P(Y=y)} = \frac{P(Y=y|X=x)P(X=x)}{\sum_{x=0}^{4} P(Y=y|X=x)P(X=x)}$$

$$P(X=3|Y=2) = \frac{P(Y=2|X=3)P(X=3)}{\sum_x P(Y=2|X=x)P(X=x)}$$

$$= \frac{0.6\cdot 0.3}{0\cdot \quad + 0\cdot \quad + 0.2\cdot 0.3 + 0.6\cdot 0.3 + 0.4\cdot 0.1}$$

$$= \frac{0.18}{0.28} = \underline{\underline{\qquad}}$$

$$P(X=3|Y\geq 2) = \frac{P(Y\geq 2|X=3)P(X=3)}{\sum_x P(Y\geq 2|X=x)P(X=x)}$$

$$= \frac{0.7\cdot 0.3}{0\cdot \quad + 0\cdot \quad + 0.2\cdot 0.3 + 0.7\cdot 0.3 + 0.9\cdot 0.1}$$

$$= \frac{0.21}{0.36} = \underline{\underline{\qquad}}$$

Oppg. 2 Sandsteinsprøver

$X \sim N(\mu_x, \sigma_x) = N(0.28, 0.08)$

a)

$$P(X > 0.30) = 1 - P(X < 0.30)$$
$$= 1 - P\left(\frac{X-\mu_x}{\sigma_x} < \frac{0.30-0.28}{0.08}\right)$$
$$= 1 - P(Z < 0.25)$$
$$= 1 - \qquad = \underline{\underline{\qquad}}$$

$$P(0.20 < X < 0.32) = P(X < 0.32) - P(X < 0.20)$$
$$= P\left(\frac{X-\mu_x}{\sigma_x} < \frac{0.32-0.28}{0.08}\right) - P\left(\frac{X-\mu_x}{\sigma_x} < \frac{0.20-0.28}{0.08}\right)$$
$$= P(Z < 0.5) - P(Z < -0.5) = \underline{\qquad}$$
$$= \qquad = \underline{\underline{\qquad}}$$

b)

$$P(X > 0.30 \mid 0.20 < X < 0.32)$$
$$= \frac{P((X > 0.30) \cap (0.20 < X < 0.32))}{P(0.20 < X < 0.32)}$$
$$= \frac{P(0.30 < X < 0.32)}{P(0.20 < X < 0.32)}$$
$$= \frac{P(Z < 0.5) - P(Z < 0.25)}{P(0.20 < X < 0.32)}$$
$$= \frac{\qquad}{\qquad} = \underline{\underline{\qquad}}$$

$$P(X < x) = 0.8 \Rightarrow P\left(Z < \frac{x-\mu_x}{\sigma_x}\right) = 0.8$$
$$\Rightarrow \frac{x-\mu_x}{\sigma_x} = z_{0.8} \Rightarrow x = \mu_x + \sigma_x z_{0.8}$$
$$x = 0.28 + 0.08 \cdot \qquad = \underline{\underline{\qquad}}$$

c) $Y = 0.4\, X_1 + 0.6\, X_2$

$Y \sim N(\mu_y, \sigma_y)$ fordi lin. komb. av normalford. variable

$\mu_y = E(Y) = 0.4 \cdot E(X_1) + 0.6 \cdot E(X_2)$

$= 0.4 \cdot 0.28 + 0.6 \cdot 0.28 = \underline{\underline{0.28}}$

$\sigma_y^2 = Var(Y) = 0.4^2\, Var(X_1) + 0.6^2\, Var(X_2)$

$= (0.4^2 + 0.6^2)\, 0.08^2 = \underline{\underline{\qquad}}$

$$Corr(Y, X_1) = \frac{Cov(Y, X_1)}{\sigma_y \cdot \sigma_x}$$

$$= \frac{1}{\sigma_y \sigma_x} Cov(0.4\, X_1 + 0.6\, X_2, X_1)$$

$$= \frac{1}{\sigma_y \sigma_x} 0.4\, Cov(X_1, X_1)$$

$$= \frac{0.4\, \sigma_x^2}{\sigma_y \sigma_x} = \frac{0.4\, \sigma_x}{\sigma_y} = \underline{\qquad} = \underline{\underline{\qquad}}$$

d)

$$\underbrace{v = c \cdot x^3}_{g(x)} \Rightarrow \underbrace{x = \frac{v^{1/3}}{c^{1/3}}}_{g^{-1}(v)}$$

$$f_V(v) = f_X(g^{-1}(v)) \left| \frac{d\, g^{-1}(v)}{dv} \right|$$

$$= \frac{1}{\sqrt{2\pi}\, \sigma_x} \cdot \exp\left(-\frac{1}{2}\left(\frac{\frac{\frac{1}{3} v^{-2/3}}{c^{1/3}} - \mu_x}{\sigma_x}\right)^2\right) \cdot \left| \frac{\frac{1}{3} v^{-\frac{2}{3}}}{c^{1/3}} \right|$$

$$= \frac{\frac{1}{3} v^{-2/3}}{\sqrt{2\pi}\, c^{1/3} \sigma_x} \cdot \exp\left(-\frac{1}{2}\left(\frac{\frac{1}{3} v^{-2/3} - c^{1/3} \mu_x}{c^{1/3} \sigma_x}\right)^2\right)$$

Oppg. 3 Bioindikatorer

$X^1 = X_{0,0.5}$

$X^2 = X_{0.5,0.25}$

a)

$X_{a,t}$ = # utslag bioindikator $[a, a+t]$

$P(X_t = x) = \frac{(\lambda t)^x}{x!} e^{-\lambda t}$; $x = 0, 1, 2, \ldots\ldots$ Poisson

samt $[X_{a,t}$ og $X_{b,s}]$ uavh hvis $[a, a+t] \cap [b, b+s] = \emptyset$

$P($tredje bioind ank i $[0.5, 0.75]t)$

$= P(X_{0,0.75} \geq 3 \cap X_{0,0.5} < 3)$

$= \sum_{i=0}^{2} P(X_{0\,0.75} \geq 3 \cap X_{0\,0.5} = i)$

$= \sum_{i=0}^{2} P(X_{0\,0.75} \geq 3 / X_{0\,0.5} = i) P(X_{0\,0.5} = i)$

$= \sum_{i=0}^{2} P(X^2 \geq 3-i / X^1 = i) P(X^1 = i)$ ← introd. X^1, X^2 som er uavh Poisson ford.

$= \sum_{i=0}^{2} [1 - P(X^2 < 3-i)] P(X^1 = i)$

$= \sum_{i=0}^{2} \left[1 - \sum_{j=0}^{3-i-1} P(X^2 = j)\right] P(X^1 = i)$

$= \sum_{i=0}^{2} \left[1 - e^{-\lambda 0.25} \sum_{j=0}^{2-i} \frac{(\lambda\, 0.25)^j}{j!}\right] \frac{(\lambda\, 0.5)^i}{i!} e^{-\lambda 0.5}$