Løysingsforslag kont ST1101 2016

Oppgåve 1

a. La $A_i \sim$ person nr i har begått forbrytelsen

$$P\left(\bigcup_{i=1}^{5000} A_i\right) = 1 \quad \text{og} \quad A_i \cap A_j = \emptyset, \quad \forall i \neq j.$$

$$\Rightarrow \quad 2\alpha + 4498\beta = 2\alpha + 4498\frac{\alpha}{c} = 1$$

$$\Rightarrow \quad \alpha = \frac{c}{2c+4498}, \qquad c = 10 \Rightarrow \alpha = \frac{10}{5018} = 1.99 \cdot 10^{-3}$$

b)

$$A_s^c \cap B_s^c = (A_s \cup B_s)^c$$

$$\Rightarrow P(A_s^c \cap B_s^c) = 1 - P(A_s \cup B_s) = 1 - P(A_s) - P(B_s) = 1 - 2\alpha$$

$$P(A_s \cap K_B^c) = P(A_s) \cdot P(K_B^c) = \alpha(1 - 10^{-5})$$

$$P(A_s \cap K_B) = P(A_s) \cdot P(K_B) = \alpha \cdot 10^{-5}$$

$$P(A_s^c \cap B_s^c \cap K_A \cap K_B) = P(A_s^c \cap B_s^c) \cdot P(K_A | A_s^c \cap B_s^c) \cdot P(K_B | A_s^c \cap B_s^c) = (1-2\alpha) \cdot 10^{-5} \cdot 10^{-5}$$
$$= \underline{(1-2\alpha)\, 10^{-10}}$$

$$P(A_s^c \cap B_s^c \cap K_A \cap K_B^c) = P(A_s^c \cap B_s^c) \cdot P(K_A | A_s^c \cap B_s^c) \cdot P(K_B^c | A_s^c \cap B_s^c)$$
$$= \underline{(1-2\alpha) \cdot 10^{-5}(1 - 10^{-5})}$$

c)

$$P(A_s | K) = \frac{P(A_s \cap K)}{P(K)} = \frac{P(A_s \cap K_B^c)}{P(A_s \cap K_B^c) + P(A_s^c \cap B_s^c \cap K_A \cap K_B^c)}$$

$$= \frac{\alpha(1 - 10^{-5})}{\alpha(1 - 10^{-5}) + (1-2\alpha) \cdot 10^{-5}(1 - 10^{-5})} = \frac{\alpha}{\alpha + (1-2\alpha) \cdot 10^{-5}}$$

$$= \frac{1}{1 + \frac{(1-2\alpha)}{\alpha \cdot 10^{5}}}, \qquad c = 10 \Rightarrow \alpha = 1.99 \cdot 10^{-3}$$

$$\Rightarrow P(A_s | K) = \frac{1}{1 + \frac{(1 - 3.98 \cdot 10^{-3})}{1.99 \cdot 10^{2}}} = 0.995$$

$D = Z_1 - Z_2 \sim N(0, 13^2 + 13^2)$ $D: N(0, 338)$

$P(|D| > 15) = 1 - P(|D| \le 15) = 1 - P(-15 \le D \le 15)$

$= 1 - P\left(\frac{-15}{\sqrt{338}} < \frac{D}{\sqrt{338}} < \frac{15}{\sqrt{338}}\right) = 1 - \left(\Phi(0.816) - \Phi(-0.816)\right) = 1 - (0.7928 - 0.2072)$

$= 1 - 0.5856 = \underline{0.4144}$

c)

$E[\hat{\mu}] = \frac{1}{12} \sum_{i=1}^{12} E[X_i] = \frac{1}{12} \sum_{i=1}^{12} \mu = \mu$

$Var[\hat{\mu}] = \frac{1}{144} \sum_{i=1}^{12} Var[X_i] = \frac{1}{144} \sum_{i=1}^{12} 12^2 = \underline{12}$

$E[\mu^*] = E\left[\frac{1}{12} \sum_{i=1}^{12} Z_i\right] - 40 = \frac{1}{12} \sum_{i=1}^{12} E[Z_i] - 40 = \frac{1}{12} \sum_{i=1}^{12} (\mu + 40) - 40 = \mu + 40 - 40 = \mu$

$Var[\mu^*] = Var\left[\frac{1}{12} \sum_{i=1}^{12} Z_i\right] = \frac{1}{144} \sum_{i=1}^{12} Var[Z_i] = \frac{1}{144} \sum_{i=1}^{12} (12^2 + 5^2) = \frac{1}{144} \cdot 12 \cdot 13^2 = \frac{13^2}{12} = \frac{169}{12}$

$= 14.08 > 12 \Rightarrow \hat{\mu}$ er best.

d)

$L[v_1, v_2, \dots, v_m; \beta] = \left(\frac{1}{\sqrt{2\pi}}\right)^m \cdot \frac{1}{\sigma^m} e^{-\frac{1}{2} \sum_{i=1}^{m} \left(\frac{v_i - \beta m_i}{\sigma}\right)^2}$

$\ln L[v_1, v_2, \dots, v_m; \beta] = \ln L(\beta) = -m \ln\sqrt{2\pi} - m \ln \sigma - \frac{1}{2} \sum_{i=1}^{m} \left(\frac{v_i - \beta m_i}{\sigma}\right)^2$

$\frac{\partial \ln L(\beta)}{\partial \beta} = 0 \Leftrightarrow \frac{1}{\sigma^2} \sum (V_i - \beta m_i) m_i = 0 \Leftrightarrow \beta \sum_{i=1}^{m} m_i^2 = \sum_{i=1}^{m} m_i V_i$

$\Rightarrow \beta_e = \frac{\sum_{i=1}^{m} m_i V_i}{\sum_{i=1}^{m} m_i^2} = \frac{\sum_{i=1}^{m} m_i v_i}{M}$ og $\hat{\beta} = \frac{\sum_{i=1}^{m} m_i V_i}{M}$

$E[\hat{\beta}] = \sum_{i=1}^{m} \frac{m_i E[V_i]}{M} = \frac{\sum_{i=1}^{m} m_i m_i \beta}{\sum_{i=1}^{m} m_i^2} = \beta$

$Var[\hat{\beta}] = \frac{\sum_{i=1}^{m} m_i^2 Var[V_i]}{M^2} = \sigma^2 \frac{M}{M^2} = \frac{\sigma^2}{M} = \underline{\frac{\sigma^2}{\sum m_i^2}}$

Oppgave 2

a) $P(X=3) = \dfrac{2^3 e^{-2}}{3!} = 0.1804$

$P(X+Y=3) = \dfrac{(2+1)^3 e^{-(2+1)}}{3!} = 0.2240$

b) $P(T_1 > t) = P($ingen hendelser i et tidsinterval av lengde $t)$

$= P(X_t = 0)$

$= \dfrac{(2t)^0 e^{-2t}}{0!} = e^{-2t}$

$P(T_1 \leq t) = 1 - e^{-2t}$

$= F_{T_1}(t)$

$f_{T_1}(t) = \dfrac{d}{dt} F_{T_1}(t) = \underbrace{2e^{-2t}}$

kjenner igjen pdf for eksponensial fordeling med $\lambda = 2$.

$E(T_1) = \dfrac{1}{\lambda} = \underline{\underline{0.5}}$

c) T = tid til første fisk.

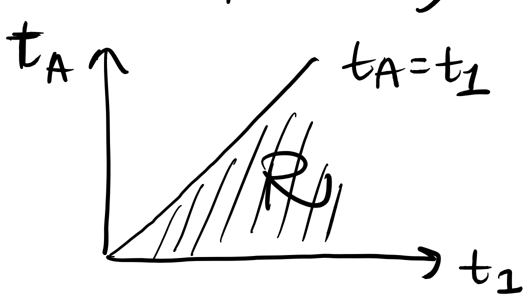
$T \sim$ eksponensial $(\lambda = 3)$

$$P(T > 1) = 1 - P(T \leq 1)$$
$$= 1 - (1 - e^{-3 \cdot 1})$$
$$= e^{-3} = \underline{\underline{0.05}}$$

T_A = tid til første abbor

T_1 = tid til første ørret

$$P(T_A < T_1) = \iint_R f_{T_A T_1}(t_A, t_1)\, dt_1\, dt_A$$



uavh.

$$= f_{T_A}(t_A)\, f_{T_1}(t_1)$$

$$= \int_0^\infty \int_0^{t_1} 2e^{-2t_1} \cdot e^{-t_A}\, dt_A\, dt_1$$

$$= \int_0^\infty 2e^{-2t_1} \left[-e^{-t_A}\right]_0^{t_1} dt_1$$

$$= \int_0^{\infty} 2e^{-2t} \left(1 - e^{-t_1}\right) dt_1$$

$$= \int_0^{\infty} 2e^{-2t_1} dt_1 - \int_0^{\infty} 2e^{-3t_1} dt_1$$

$$= \left[-e^{-t_1}\right]_0^{\infty} - \left[-\frac{2}{3}e^{-3t_1}\right]_0^{\infty}$$

$$= (1 - 0) - \left(\frac{2}{3} - \frac{2}{3}\cdot 0\right)$$

$$= 1 - \frac{2}{3} = \underline{\underline{\frac{1}{3}}}$$

Oppgave 3

a) $P(X < 490) = P\left(Z \leq \frac{490 - 511}{12}\right)$

$= \Phi(-1.75) = 0.0401$

$U \sim$ binomisk $(12, 0.0401)$

$P(U \geq 1) = 1 - P(U = 0) =$

$1 - \binom{12}{0} 0.0401^0 (1 - 0.0401)^{12}$

$= 0.3881$

b) X = vekt porsjon pulver

Y = vekt eske

$X+Y \sim N(511+40,\ 12^2+5^2)$

$X+Y \sim N(551,\ 13^2)$

$P(X+Y > 535) = 1 - P(X+Y \leq 535)$

$= 1 - P\left(Z \leq \frac{535-551}{13}\right)$

$= 1 - \Phi(-1.23) = 0.8907$

V_1 = vekt eske + pulver 1

V_2 = — " — 2

$P(|V_1 - V_2| > 15) =$

$P(V_1 - V_2 \leq -15) + P(V_1 - V_2 \geq 15)$

Obs: $V_1 - V_2 \sim N(0,\ 2 \cdot 13^2)$

$= 2P(V_1 - V_2 \leq -15) = 2P\left(Z \leq \frac{-15}{\sqrt{2} \cdot 13}\right)$

$= 2\Phi(-0.82) = 2 \cdot 0.2061 = 0.4122$