Løsningsforslag til ST1101 / ST6200 Sannsynlighetsregning
Torsdag 7. juni 2007

Oppgave 1

a) $E(Y) = \int_0^\infty yke^{-y/\lambda}dy = k\int_0^\infty y^1 e^{-(1/\lambda)y}dy = \underline{\underline{k\lambda^2}}$

$E(Y^2) = k\int_0^\infty y^2 e^{-(1/\lambda)y}dy = \underline{\underline{2k\lambda^3}}$

$Var(Y) = E(Y^2) - (E(Y))^2 = 2k\lambda^3 - k^2\lambda^4 = \underline{\underline{k\lambda^3(2-k\lambda)}}.$

b) $F_Y(y) = \int_0^y ke^{-y/\lambda}dy = \underline{\underline{k\lambda(1-e^{-y/\lambda})}}$, for $y \geq 0$

$$P\left(\frac{1}{4}\mu \leq Y \leq \frac{3}{4}\mu\right) = F\left(\frac{3}{4}\mu\right) - F\left(\frac{1}{4}\mu\right) = k\lambda\left(1-e^{-\frac{3\mu}{4\lambda}}\right) - k\lambda\left(1-e^{-\frac{\mu}{4\lambda}}\right) = \underline{\underline{k\lambda\left(e^{-\frac{\mu}{4\lambda}} - e^{-\frac{3\mu}{4\lambda}}\right)}}$$

c) $E(\bar{Y}) = E(Y) = \underline{\underline{k\lambda^2}}$, $Var(\bar{Y}) = \frac{1}{20}Var(Y) = \underline{\underline{\frac{k\lambda^3}{20}(2-k\lambda)}}$

$\int_0^\infty ke^{-y/\lambda}dy = k\lambda = 1 \Rightarrow k = \underline{\underline{\frac{1}{\lambda}}}$

$$P(\bar{Y} \geq 1.2\mu) = P\left(\frac{\bar{Y}-\lambda}{\lambda/\sqrt{20}} \geq \frac{1.2\lambda-\lambda}{\lambda/\sqrt{20}}\right) \approx P(z \geq 0.2\sqrt{20}) = P(z \geq 0.8944) \approx \underline{\underline{0.19}}$$

Oppgave 2

a) $X \sim Po(\alpha/3) \Rightarrow E(X) = \alpha/3 = \underline{\underline{8/3}}$

$$P(3 < X < 7) = P(X=4) + P(X=5) + P(X=6) = \frac{e^{-8/3}(8/3)^4}{4!} + \ldots = \underline{\underline{0.26}}$$

b) Y er eksponensialfordelt med parameter $\lambda = 1/3$.
$T = 12\times 5 + 12\times Y \Rightarrow E(T) = 60 + 12\times E(Y) = 60 + 12\times 3 = \underline{\underline{96}}$

$Var(T) = 12^2 Var(Y) = 12^2 \times 3^2 \Rightarrow SD(T) = 12\times 3 = \underline{\underline{36}}$

c) $H \mid x \sim Bin(x,p), \quad E(H \mid x) = xp$

$$E(H) = E_x\left[E(H \mid x)\right] = E_x(xp) = pE_x(x) = \underline{\underline{\frac{p\alpha}{3}}}$$

$$f_H(h) = P(H=h) = \sum_x P(H=h, X=x) = \sum_x P(H=h \mid X=x)P(X=x)$$

$$= \sum_x \binom{x}{h} p^h (1-p)^{x-h} \frac{e^{-\alpha/3}(\alpha/3)^x}{x!} = \frac{e^{-\alpha/3}p^h(\alpha/3)^h}{h!} \sum_{x=h}^{\infty} \frac{\left[\alpha/3(1-p)\right]^{x-h}}{(x-h)!}$$

Bruker så $y = x - h$ og at $\sum_{y=0}^{\infty} \frac{a^y}{y!} = e^a$

$\Rightarrow f_H(h) = \frac{e^{-p\alpha/3}(p\alpha/3)^h}{h!}$. Dvs. Poissonfordeling med parameter $p\alpha/3$.

Oppgave 3

La A = Dobbel 6 $\Rightarrow P(A) = \left(\frac{1}{6}\right)^2 = \frac{1}{36} \quad og \quad P(\overline{A}) = \frac{35}{36}$

Uavhengige kast $\Rightarrow$ *P*(Ikke A i løpet av *n* kast)= $\left(\frac{35}{36}\right)^n$

$\Rightarrow$ *P*(minst en A i løpet av *n* kast) = $1 - \left(\frac{35}{36}\right)^n$

Setter inn for *n* og finner at vi for *n*=24 får 0.49, men for *n*=25 får 0.506, så $\underline{\underline{n=25}}$

Oppgave 4

a) $X \sim N(m, 0.01^2)$

$$m = 1.00 \Rightarrow P(X < 0.99) = P\left(\frac{X-1}{0.01} < \frac{0.99-1}{0.01}\right) = P(Z < -1) = \underline{\underline{0.1587}}$$

b) $P(X < 0.99) = p = P(undervektig)$

P(ingen undervektige)= $(1-p)^{20}$

P(minst en undervektig) $= 1-(1-p)^{20} < 0.01$

$(1-p)^{20} > 0.99 \Rightarrow p = 0.0005$

Fra Tab. A1: $\frac{0.99-m}{0.01} = -3.3 \Rightarrow m = \underline{\underline{1.023}}$