Løysingsforslag ST6101 16/5 - 2013

Oppgåve 1

a) X = sluttida i minutt, $X \sim N(230, 25^2)$

$$P(X \leq 165) = P\left(\frac{X-230}{25} \leq \frac{165-230}{25}\right) = \Phi\left(-\frac{65}{25}\right) = \Phi(-2.6) = \underline{0.0047}$$

$$P(X > 270) = P\left(\frac{X-230}{25} > \frac{270-230}{25}\right) = 1 - \Phi(1.6) = 1 - 0.9452 = \underline{0.0548}$$

b) La $Y = \sum_{i=1}^{5} X_i$, $E[Y] = \sum_{i=1}^{5} E[X_i] = 230 \cdot 5 = 1150$

$$Var[Y] = \sum_{i=1}^{5} Var[X_i] = 625 \cdot 5 = 3125$$

$$P(Y > 1100) = P\left(\frac{Y-1150}{\sqrt{3125}} > \frac{1100-1150}{\sqrt{3125}}\right) = 1 - \Phi\left(\frac{-50}{\sqrt{3125}}\right) = 1 - \Phi(-0.894)$$
$$= \underline{0.8133}$$

$$P(X_1 \leq 230 \cap X_2 \leq 230 \cap \cdots \cap X_5 \leq 230) = \prod_{i=1}^{5} P(X_i \leq 230) = \underline{\left(\tfrac{1}{2}\right)^5} = \underline{\tfrac{1}{32}}$$

Oppgåve 2

a) 20 uavh. forsøk

Reg: Defekt / Ikkje defekt

$P(\text{Defekt}) = p$ i kvart forsøk

$\Rightarrow X \sim \text{Binomisk}(20, p)$

$$P(X > 6) = 1 - P(X \leq 6) \underset{\text{tabell}}{\overset{n=20,\, p=0.4}{=}} 1 - 0.25 = 0.75$$

Y = talet på skyttarar som har fått esker med meir enn 6 defekte patroner. $Y \sim B(10, 0.75)$

$$P(Y = 4) = P(10 - Y = 6) = P(Z = 6) \quad \text{der} \quad Z \sim B(10, 0.25)$$

$$P(Z = 6) \overset{\text{~~frå tabell~~}}{=} \binom{10}{6}(0.25)^6(0.75)^4 \overset{\text{frå tabell}}{=} 0.996 - 0.980 = \underline{0.016}$$

b) La S være $\sim B(19, 0.4)$

$P(X \geq 5 \mid$ gitt den første er defekt$) = P(S \geq 4)$ sidan patronane er defekte uavhengige av om andre er defekte.

$P(S \geq 4) = 1 - P(S \leq 3) = 1 - 0.023 = \underline{0.977}$

La D være talet på defekte som blir trekte ut.

$$P(D=2) = \frac{\binom{5}{2}\binom{15}{3}}{\binom{20}{5}} = \frac{5!}{2!\cdot 3!} \cdot \frac{15!}{3!\cdot 12!} \cdot \frac{5!\cdot 15!}{20!} = \frac{2285}{7752} = \underline{0.293}$$

c)

$$L(p \mid w_1, w_2, \ldots, w_5) = \prod_{i=1}^{5} P(W_i = w_i) = \prod_{i=1}^{5} \binom{m_i}{w_i} p^{w_i} (1-p)^{m_i - w_i}$$

$$= \left(\prod_{i=1}^{5} \binom{m_i}{w_i}\right) p^{\sum_{i=1}^{5} w_i} \cdot (1-p)^{\sum m_i - \sum w_i} = k \cdot p^{\sum_{i=1}^{5} w_i} \cdot (1-p)^{\sum_{i=1}^{5} m_i - \sum_{i=1}^{5} w_i}$$

$$\ln L(\cdot) = \ln k + \sum_{i=1}^{5} w_i \ln p + \left(\sum_{i=1}^{5} m_i - \sum_{i=1}^{5} w_i\right) \ln(1-p)$$

$$\frac{\partial \ln L}{\partial p} = \frac{1}{p} \sum w_i - \left(\sum_{i=1}^{5} m_i - \sum_{i=1}^{5} w_i\right) \cdot \frac{1}{1-p} = 0$$

$$\Leftrightarrow (1-p) \sum_{i=1}^{5} w_i - p \sum_{i=1}^{5} m_i + p \sum_{i=1}^{5} w_i = 0 \Rightarrow p_e = \frac{\sum_{i=1}^{5} w_i}{\sum_{i=1}^{5} m_i} = \frac{\sum_{i=1}^{5} w_i}{80}$$

Sidan $\frac{d \ln L}{dp}$ går frå $+$ til $-$ når p aukar.

$$SSMP = \hat{p} = \frac{\sum_{i=1}^{5} W_i}{\sum_{i=1}^{5} m_i} = \underline{\frac{\sum_{i=1}^{5} W_i}{80}}$$

$$p_e = \frac{37}{80} \approx 0.46.$$

d) $E[\hat{p}] = \dfrac{\sum_{i=1}^{5} E[W_i]}{80} = \dfrac{20p \cdot 3 + 10p \cdot 2}{80} = \dfrac{80p}{80} = \underline{p}$

$Var[\hat{p}] = \dfrac{\sum_{i=1}^{5} Var[W_i]}{80^2} = \dfrac{3 \cdot 20p(1-p) + 2 \cdot 10p(1-p)}{80^2} = \underline{\dfrac{p(1-p)}{80}}$

$$P\left(-1.96 < \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{80}}} < 1.96\right) \approx 0.95$$

$$\Leftrightarrow P\left(\hat{p} - 1.96\sqrt{\frac{p(1-p)}{80}} < p < \hat{p} + 1.96\sqrt{\frac{p(1-p)}{80}}\right) \approx 0.95$$

Erstattar p med $\hat{p}$ og får tilnærma konfidensintervall

$$\left(\hat{p} - 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{80}},\ \hat{p} + 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{80}}\right) = (0.349,\ 0.571)$$

Oppgåve 3

a)

$P(X \le 10) \overset{\text{frå tabell}}{=} 0.583$

$P(4 \le X \le 10) = P(X \le 10) - P(X \le 3) = 0.583 - 0.0103 = \underline{0.5727}$

b) $M_X(t) = E[e^{tX}] = \sum_{x=0}^{\infty} \dfrac{e^{tx} \lambda^x e^{-\lambda}}{x!} = e^{-\lambda} \sum_{x=0}^{\infty} \dfrac{(\lambda e^t)^x}{x!} = e^{-\lambda} e^{\lambda e^t} = \underline{e^{\lambda(e^t-1)}}$

$M_X'(t) = e^{\lambda(e^t-1)} \cdot \lambda e^t. \quad M_X'(0) = \lambda = \underline{E[X]}$

$M_X''(t) = e^{\lambda(e^t-1)} \cdot (\lambda e^t)^2 + \lambda e^t e^{\lambda(e^t-1)}$

$M_X''(0) = \lambda^2 + \lambda = E[X^2]$

$Var[X] = E[X^2] - (E[X])^2 = \lambda^2 + \lambda - \lambda^2 = \underline{\lambda}$

Oppgåve 4

La $F \sim$ Flyet blir funne

$R_i \sim$ Flyet er i region i

$M \sim$ Søk i region 1 er mislukka

a)
$$P(F) = P(F \cap R_1) + P(F \cap R_2) + P(F \cap R_3)$$
$$= P(F|R_1) \cdot P(R_1) + P(F|R_2) \cdot P(R_2) + P(F|R_3) \cdot P(R_3)$$
$$= \beta_1 \cdot \frac{1}{3} + \beta_2 \cdot \frac{1}{3} + \beta_3 \cdot \frac{1}{3} = \frac{1}{3}(\beta_1 + \beta_2 + \beta_3)$$

$$P(M) = P(M \cap R_1) + P(M \cap R_2) + P(M \cap R_3)$$
$$= P(M|R_1) \cdot P(R_1) + P(M|R_2) \cdot P(R_2) + P(M|R_3) \cdot P(R_3)$$
$$= (1-\beta_1) \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} + 1 \cdot \frac{1}{3} = 1 - \frac{\beta_1}{3}$$

b)
$$P(R_1|M) = \frac{P(R_1 \cap M)}{P(M)} = \frac{P(M|R_1) \cdot P(R_1)}{P(M)} = \frac{(1-\beta_1) \cdot \frac{1}{3}}{1 - \frac{\beta_1}{3}}$$
$$= \frac{1-\beta_1}{3-\beta_1} < \frac{1}{3}$$ ⟹: Sannsynet har minka.

$$P(R_2|M) = \frac{P(R_2 \cap M)}{P(M)} = \frac{P(M|R_2) \cdot P(R_2)}{P(M)} = \frac{1 \cdot \frac{1}{3}}{1 - \frac{\beta_1}{3}} = \frac{1}{3-\beta_1} > \frac{1}{3}$$

⟹: Sannsynet har auka.

$$P(R_3|M) = P(R_2|M)$$