# Løysingsforslag ST1101, Vår 2017



## Oppgåve 1

a) B og C er uavh. dersom $P(B \cap C) = P(B) \cdot P(C)$

$$P(A) = \frac{1}{3} \cdot \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{4} \cdot \frac{1}{10} \cdot \frac{1}{1000} = \frac{1}{12} \cdot 10^{-6}$$

b) Vi får
$$\left.\begin{array}{l} N \text{ uavh. forsøk} \\ \text{Reg } A/A^c \\ P(A) = p \end{array}\right\} \Rightarrow \begin{array}{l} P(X=k) = \binom{N}{k} p^k (1-p)^{N-k} \\ \text{eller } X \sim B(N,p) \end{array}$$

$$P(X=1) = \binom{N}{1} p (1-p)^{N-1} = Np(1-p)^{N-1}$$

$$P(X \geq 1) = 1 - P(X=0) = 1-(1-p)^N$$

c)
$$P(X>1 \mid X \geq 1) = \frac{P(X>1 \cap X \geq 1)}{P(X \geq 1)} = \frac{P(X>1)}{P(X \geq 1)} = \frac{1-P(X \leq 1)}{1-P(X=0)}$$

$$= \frac{1-P(X=0)-P(X=1)}{1-P(X=0)} = \frac{1-(1-p)^N - Np(1-p)^{N-1}}{1-(1-p)^N}$$

N stor og p liten $\Rightarrow P(X=k) \approx \frac{\lambda^k e^{-\lambda}}{k!}$ der $\lambda = Np = \frac{1}{12} 10^{-6} \cdot 6 \cdot 10^6 = \underline{\frac{1}{2}}$

$$\Rightarrow P(X>1 \mid X \geq 1) \approx \frac{1-e^{-\frac{1}{2}} - \frac{1}{2}e^{-\frac{1}{2}}}{1-e^{-\frac{1}{2}}} = \frac{1-\frac{3}{2}e^{-\frac{1}{2}}}{1-e^{-\frac{1}{2}}} = \frac{1-0.9098}{1-0.6065} = \underline{0.23}$$

# Oppgave 2



a)

$$P(X>0) = P\left(\frac{X-0.0165}{0.073} > \frac{0-0.0165}{0.073}\right) = P(Z > -0.226) = 1 - \Phi(-0.226)$$

$$= 1 - 0.4105 = \underline{0.5895}$$

$$P(-0.1 < X < 0.1) = P\left(\frac{-0.1-0.0165}{0.073} < \frac{X-0.0165}{0.073} < \frac{0.1-0.0165}{0.073}\right)$$

$$= P\left(-\frac{0.1165}{0.073} < Z < \frac{0.0835}{0.073}\right) = \Phi(1.144) - \Phi(-1.596) = 0.8738 - 0.0553 = \underline{0.8185}$$

b)

$$P(Y_1 > 1) = P(\ln(Y_1) > 0) = P(X > 0) = \underline{0.5895}$$

$$P\left(\frac{S(2)}{S(0)} > 1\right) = P\left(\ln\left(\frac{S(2)}{S(1)} \cdot \frac{S(1)}{S(0)}\right) > 0\right) = P\left(\ln\left(\frac{S(2)}{S(1)}\right) + \ln\left(\frac{S(1)}{S(0)}\right) > 0\right)$$

$$= P(\ln(Y_1) + \ln(Y_2)) > 0$$

$$E[\ln(Y_1) + \ln(Y_2)] = 2 \cdot 0.0165 = 0.033$$

$$Var[\ln(Y_1) + \ln(Y_2)] = 2 \cdot (0.073)^2 \Rightarrow SD[\ln Y_1 + \ln Y_2] = \underline{\sqrt{2} \cdot 0.073}$$

$\Rightarrow$: $V = \ln(Y_1) + \ln(Y_2) \sim N(0.033, 2 \cdot (0.073)^2)$

$$P(V > 0) = P\left(\frac{V - 0.033}{0.073 \cdot \sqrt{2}} > -\frac{0.033}{0.073\sqrt{2}}\right) = P\left(Z > \frac{-0.033}{0.073 \cdot \sqrt{2}}\right)$$

$$= 1 - P(Z \leq -0.32) = 1 - \Phi(-0.32) = 1 - 0.3745 = \underline{0.6255}$$

c)

$$X = \ln Y \Rightarrow Y = e^X$$

$$E[Y] = M_X(1) = e^{\mu + \frac{1}{2}\sigma^2} = e^{0.0192} = 1.02.$$

$$E[Y^2] = M_X(2) = e^{2\mu + 2\sigma^2}$$

$$\Rightarrow Var[Y] = e^{2\mu + 2\sigma^2} - e^{2\mu + \sigma^2} = e^{2\mu + \sigma^2}(e^{\sigma^2} - 1)$$

$$= e^{0.038} \cdot (e^{0.005} - 1) = \underline{0.0055}$$

Oppgåve 3

a) Dersom T skal bli lik k, må vi i tillegg til den 1. tørre dagen ha k-1 dagar der vêret ikkje forandrar seg og så må det skifte på den k-te dagen (1. dagen ikkje medrekna). Sannsynet for dette blir $(1-p)^{k-1}\cdot p$ p.g.a. uavh. Derfor er

$$P(T=k) = (1-p)^{k-1}\cdot p, \quad k=1,2,\ldots$$

Dette er ei geometrisk fordeling.

$$P(T\le k) = 1-P(T>k) = 1-(1-p)^k$$

b)
$$M_T'(t) = pe^t\cdot\frac{1}{1-(1-p)e^t} + \frac{(1-p)e^t\,pe^t}{(1-(1-p)e^t)^2} = \frac{pe^t}{(1-(1-p)e^t)^2}$$

$$M_T''(t) = pe^t\cdot\frac{1}{(1-(1-p)e^t)^2} + \frac{2(1-p)e^t\,pe^t}{(1-(1-p)e^t)^3} = \frac{pe^t(1+(1-p)e^t)}{(1-(1-p)e^t)^3}$$

$$E[T] = M_T'(0) = \frac{p}{p^2} = \frac{1}{p}$$

$$Var[T] = M_T''(0) - (M_T'(0))^2 = \frac{2-p}{p^2} - \frac{1}{p^2} = \frac{1-p}{p^2}$$

c)
$$L(p) = \prod_{i=1}^{m}(1-p)^{k_i-1}\cdot p = p^m(1-p)^{\sum k_i - m}$$

$$\ln L(p) = m\ln p + \left(\sum_{i=1}^{m}k_i - m\right)\ln(1-p)$$

$$\frac{d\ln L(p)}{dp} = 0 \iff \frac{m}{p} - \frac{\left(\sum_{i=1}^{m}k_i - m\right)}{1-p} = 0 \Rightarrow m - pm - p\sum_{i=1}^{m}k_i + pm = 0$$

$$\Rightarrow p_e = \frac{m}{\sum_{i=1}^{m}k_i} = \frac{1}{\bar{k}}$$

SSME: $\hat{p} = \frac{1}{\bar{T}}$

d)

$E[T] = \frac{1}{p} \Rightarrow E[\bar{T}] = \frac{1}{p} \Rightarrow$ stor $p$ gjer at $\bar{T}$ er forventa å bli liten.

$H_0: \; p = 0.3 \qquad H_1: \; p > 0.3$

Teststatistikk $\bar{T} \approx N\left(\frac{1}{p}, \frac{1-p}{np^2}\right)$ som under $H_0$ med $n = 100$

blir $\bar{T} \approx N\left(\frac{10}{3}, \frac{7}{90}\right)$

Forkastar $H_0$ når $\bar{T}$ er liten eller når

$$Z = \frac{\bar{T} - \frac{10}{3}}{\sqrt{\frac{7}{90}}} \le -z_{0.05} = -1.645$$

$$z_{obs} = \frac{2.9 - \frac{10}{3}}{\sqrt{\frac{7}{90}}} = -1.55 > -1.645 \Rightarrow$$ ingen grunn til å

forkaste $H_0$ på 5% nivå.