Norges teknisk–
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English

Contact during exam:
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EXAM IN ST1201 STATISTICAL METHODS

Tuesday December 20 2005
Time: 09:00–13:00

Aids: All typed and written.
All calculators allowed.

Grading: January 17 2006.

Oppgave 1

A research institute has five different types of instruments for measuring the amount of infrared radiation. An experiment is done to check whether the different instruments give similar measurements. For each of fifteen objects the amount of infrared radiation is measured with each of the five instruments. The fifteen objects used all differ in terms of type of material, temperature and size.

A (partially finished) analysis of variance (ANOVA) table for the measurements reads as follows.

Source	df	SS	MS	F	p-value
Instrument	$\star$	$\star$	$\star$	$\star$	0.000
Object	$\star$	45.48159	$\star$	$\star$	0.000
Error	$\star$	0.26981	$\star$		
Total	$\star$	45.95359			

a) What design of experiment is used in the situation described above?

Write down the complete ANOVA table. In particular, show how you obtain the values for the positions with $\star$ in the above table.

Specify the stochastic model corresponding to the above ANOVA table. In particular, give interpretations for the various model parameters in terms of the situation discussed above.

b) Two p-values are given in the ANOVA table. Specify the null hypotheses, H_0, corresponding to each of these two p-values.

Which of the two p-values are of interest for the research institute? What is the conclusion of the corresponding test? (Give reasons for the answers!)

Oppgave 2

Let $X_1, X_2, \ldots, X_n$ be a random sample from a gamma distribution with parameters r and λ, i.e. with probability density function

$$f(x; r, \lambda) = \frac{\lambda^r}{\Gamma(r)} x^{r-1} e^{-\lambda x}, \;\; x > 0, r > 0, \lambda > 0.$$

It is given that the mean and variance in a gamma distribution are

$$\mathrm{E}[X] = \frac{r}{\lambda} \quad \text{and} \quad \mathrm{Var}[X] = \frac{r}{\lambda^2},$$

respectively.

Work out the moment estimators for r and λ based on $X_1, X_2, \ldots, X_n$.

Oppgave 3

Let $X_1, X_2, \ldots, X_n$ be a random sample from a normal distribution with (known) mean value $\mathrm{E}[X_i] = 1$ and (unknown) variance $\mathrm{Var}[X_i] = \theta$.

a) Show that the maximum likelihood estimator (MLE) for θ is

$$\widehat{\theta} = \frac{1}{n} \sum_{i=1}^{n} (X_i - 1)^2.$$

Explain why $n\widehat{\theta}/\theta$ has a χ^2 distribution with n degrees of freedom.

b) Show also that $\widehat{\theta}$ is unbiased and find the estimator variance. *Hint: Use what you know from* **a**), *i.e. that* $n\widehat{\theta}/\theta \sim \chi^2_n$.

Use Cramer-Rao's inequality to show that $\widehat{\theta}$ is a *best* estimator, i.e. show that there does not exist unbiased estimators for θ with variance less than that of $\widehat{\theta}$.

c) Work out a $(1 - \alpha) \cdot 100\%$ confidence interval for θ.

One also wants to use the observed values for $X_1, X_2, \ldots, X_n$ to test

$$\mathrm{H}_0 : \theta = 1 \quad \text{against} \quad \mathrm{H}_1 : \theta \neq 1.$$

d) Using $\widehat{\theta}$ as the test statistic, work out a decision rule for when to reject H_0. Use significance level α.

Work out the power function for the test. Make a rough sketch of the power function when $\alpha = 0.05$ and $n = 10$.

e) Find the generalised likelihood ratio, λ, for the hypotheses H_0 and H_1 given above.

Explain why the test you worked out in **d**) is not a generalised likelihood ratio test (GLRT).