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Contact:
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ST1201/ST6201 STATISTICAL METHODS

Friday 14 December 2012
Time: 09:00–13:00

Permitted material:
Statistiske tabeller og formler, Tapir forlag.
K.Rottman. Matematisk formelsamling.
Yellow A4-sheet of paper with handritten notes.
Calculator.

Results: 11 January 2013

Oppgave 1

Let $X_1, ..., X_{100}$ be a random sample from a normal distribution with unknown expectation μ and variance $\sigma^2 = 25$. The hypothesis $H_0 : \mu = 0$ is tested against $H_1 : \mu > 0$ (H_0 is rejected for large values of $\bar{X}$). For $\mu = 1$ the power of the test is $1 - \beta(1) = 0.5$.

a) What does the significance level α equal?

b) Find the power $1 - \beta(2)$ for $\mu = 2$.

Oppgave 2

Two independent samples of sizes $n = 200$ and $m = 240$ are taken from normal distributions with unknown expectations μ_X, μ_Y and known variances $\sigma_X^2 = 1$ and $\sigma_Y^2 = 1.2$, respectively. $H_0 : \mu_X = \mu_Y$ is being tested against $H_1 : \mu_X \neq \mu_Y$.

a) Find the $P-$value if observed sample means are $\bar{x} = 2.1$ and $\bar{y} = 2.0$.

Oppgave 3

The following result is well-known.

A. If the random vector (X, Y) has a bivariate normal distribution, and X, Y are uncorrelated (the correlation coefficient $\rho(X, Y) = 0$), then X and Y are independent.

Consider the following example. Let X and T be independent random variables, X has the standard normal distribution, T takes on two values -1 and 1, each with probability 1/2. Let $Y = TX$.

a) Show that Y has a normal distribution and therefore both components X and Y of the bivariate random vector (X, Y) are normal.

b) Show that $\rho(X, Y) = 0$ but X and Y are dependent.

c) Explain, why the example of this problem is not in contradiction with proposition A.

Oppgave 4

A researcher would like to find out (using ANOVA technique) whether a woman's name affects hers weight. The data (weights of 12 women) are given in the table.

Anna	Elsa	Julia
67	53	63
48	61	69
50	72	51
52	75	54

a) Show the ANOVA table (without "$P-$value"-column).

b) Test whether the differences among the average weights are statistically significant. The significance level $\alpha = 0.05$.

Oppgave 5

The data, presented in the Table, are 16 independent observations from a continuous, symmetric (about the unknown expectation μ) distribution. Test the hyposesis $H_0 : \mu = 0.5$ versus $H_1 : \mu > 0.5$ (significance level $\alpha = 0.05$),

0.57	0.84	0.61	0.39	0.42	0.71	0.28	0.32
0.63	0.51	0.48	0.82	0.69	0.77	0.53	0.56

a) using the large-sample sign test;

b) using the large-sample Wilcoxon signed rank test.