Eksamen i Desember 2020

Oppgave 3

Kor leis utviklingstid for dronningbier avhenger av temperaturen.

$$Y_{ij} = \mu_j + \varepsilon_{ij} \begin{cases} N(0,\sigma^2) \text{ og uavh} \\ j = 1,2,3. \quad i = 1,2,\dots,n_j \end{cases}$$

Temperatur	n_j	$\bar{Y}_{\cdot j}$	S_j
Lav (31°C)	14	14.5	0.45
Middels (32.5°C)	14	15.1	0.48
Høy (34°C	16	15.5	0.42

Vi har: $S_j^2 = \dfrac{\sum_{i=1}^{n_j} (Y_{ij} - \bar{Y}_{\cdot j})^2}{n_j - 1} \Rightarrow (n_j - 1) S_j^2 = \sum_{i=1}^{n_j} (Y_{ij} - \bar{Y}_{\cdot j})^2$

$$SSE = \sum_{j=1}^{3} \sum_{i=1}^{n_j} (Y_{ij} - \bar{Y}_{\cdot j})^2 = \sum_{j=1}^{3} (n_j - 1) S_j^2$$

$$= 13 \cdot (0.45)^2 + 13 \cdot (0.48)^2 + 15 \cdot (0.42)^2 = 8.27$$

$$SST = \sum_{j=1}^{3} n_j (\bar{Y}_{\cdot j} - \bar{Y}_{\cdot\cdot})^2$$

$$\bar{Y}_{\cdot\cdot} = \frac{Y_{\cdot 1} + Y_{\cdot 2} + Y_{\cdot 3}}{14 + 14 + 16} = \frac{14.5 \cdot 14 + 15.1 \cdot 14 + 15.5 \cdot 16}{14 + 14 + 16} = 15.05$$

slik at $SST = 14(0.55)^2 + 14(0.05)^2 + 16(0.45)^2 = 7.51$

$$SSTOT = 8.27 + 7.51 = 15.78 = \sum_{j=1}^{3} \sum_{i=1}^{n_j} (Y_{ij} - \bar{Y}_{\cdot\cdot})^2$$

Variansanalyse tabellen

Kilder	df.	SS	MS	F
Temperatur	2	7.51	3.755	$\frac{3.755}{0.202} = 18.59$
Feil	41	8.27	0.202	
Total	43	15.78		

H_0: $\mu_1 = \mu_2 = \mu_3$ $\quad$ H_1: minst 2 forskjellige

$F_{0.95,\,2,\,41} = 3.22 < F_{obs} = 18.54 \Rightarrow$ forkaster H_0 på nivå $\alpha = 0.05$

b)

H_0: $\mu_2 = \frac{\mu_1 + \mu_3}{2}$

$C = -\frac{1}{2}\mu_1 - \frac{1}{2}\mu_3 + \mu_2$ $\quad$ $H_0: C = 0$, $\quad$ $H_1: C \neq 0$,

$\hat{C} = -\frac{1}{2}\bar{Y}_{\cdot 1} - \frac{1}{2}\bar{Y}_{\cdot 3} + \bar{Y}_{\cdot 2}$

$$SS_C = \frac{\hat{C}^2}{\sum_{j=1}^{k} \frac{c_j^2}{n_j}} = \frac{\hat{C}^2}{\frac{1}{4} + \frac{1}{16}}$$

$$= \frac{\hat{C}^2}{\frac{1}{4}\cdot\frac{1}{14} + \frac{1}{4}\cdot\frac{1}{16} + \frac{1}{14}} = \frac{\hat{C}^2}{\frac{47}{64.7}}$$

$\hat{C} = -\left(\frac{14.5 + 15.5}{2}\right) + 15.1 = 0.1 \Rightarrow SS_C = \frac{(0.1)^2 \cdot 64.7}{47} = 0.095$

$$F = \frac{\frac{SS_C}{1}}{\frac{SSE}{41}} \sim F_{1,41}: \quad F_{obs} = \frac{0.095}{0.202} = 0.472 < F_{0.95,\,1,\,41} = 4.07$$

$\Rightarrow$ ingen grunn til å forkaste H_0 på nivå $\alpha = 0.05$

Oppgave 4a

Fysisk undv.	Digital undv.	D = X - Y
X_1	Y_1	D_1
⋮		⋮
X_{16}	Y_{16}	D_{16}

a) Parbasert.

$D_1, \dots, D_{16} \sim N(\mu_D, \sigma_D^2)$ og uavh.

$H_0: \mu_D = 0$ $\quad H_1: \mu_D \neq 0$

$$T = \frac{\bar{D}}{\frac{S_D}{\sqrt{16}}} = \frac{4\bar{D}}{S_D} \sim t\text{-fordelt med 15 frihetsgrader}$$

$$\bar{d}_{obs} = \frac{\sum_{i=1}^{16} d_i}{16} = \frac{80}{16} = 5$$

$$s_{D\,obs}^2 = \frac{\sum_{i=1}^{16} (d_i - \bar{d})^2}{15} = \frac{\sum_{i=1}^{16} (d_i^2 - 2d_i\bar{d} + \bar{d}^2)}{15}$$

$$= \frac{\sum_{i=1}^{16} d_i^2 - 2\bar{d}\sum_{i=1}^{16} d_i + 16\bar{d}^2}{15} = \frac{\sum_{i=1}^{16} d_i^2 - 32\bar{d}^2 + 16\bar{d}^2}{15}$$

$$= \frac{\sum_{i=1}^{16} d_i^2 - 16\bar{d}^2}{15} = \frac{1}{15}\left(1604 - 16 \cdot 5^2\right) = \frac{1204}{15} = 80.27$$

$$T_{obs} = \frac{20}{\sqrt{80.27}} = 2.23 > t_{0.025,\,15} = 2.131 \Rightarrow \text{forkast } H_0$$

på nivå $\alpha = 0.05$