(1)

	Julekuler	Mus	Figur	Karamell	Cars
Expected	$300\cdot 0{,}26 = 78$	$300\cdot 0{,}14 = 42$	$300\cdot 0{,}24$ 72	$300\cdot 0{,}2$ 60	$300\cdot 0{,}16$ 48
Observed	100	47	62	53	38

=> Goodness-of-test (parameters known)

$$\chi^2 = \frac{(100-78)^2}{78} + \frac{(47-42)^2}{42} + \frac{(62-72)^2}{72} + \frac{(53-60)^2}{60} + \frac{(38-48)^2}{48}$$

$$= 11{,}089.$$

$$\chi^2_{0.05,4} = 9{,}488$$

=> We reject the null hypothesis that the actual distribution of the "favorites" follows the proportion given by the chocolate company.

2

a)

$$\hat{\beta}_1 = \frac{n \sum_{i=1}^{n} x_i Y_i - (\sum x_i)(\sum Y_i)}{n (\sum x_i^2) - (\sum x_i)^2}$$

$$= \frac{10 \cdot 997.223 - 235.8 \cdot 41.78}{10 \cdot 5600.44 - (235.8)^2}$$

$$\approx 0.29$$

$$\hat{\mu} = \bar{Y} - \beta_1 \bar{x} = -2.877$$

b) $\hat{Y}_4 = \hat{\mu} + \hat{\beta} \cdot x_4 \approx 1{,}79038$

$$s^2 = \frac{1}{(n-2)} \cdot \left(\sum y_i^2 - \mu \cdot \sum y_i - \beta \cdot \sum xy \right)$$

$$\approx 0.208$$

$\Rightarrow$ 95% CI

$$(\hat{Y}_4 - w, \hat{Y}_4 + w)$$

$$w = t_{1-\alpha/2, n-2} \cdot s \sqrt{\frac{1}{n} + \frac{(x - \bar{x})^2}{\underbrace{\sum (x_i - \bar{x})^2}_{40{,}270}}}$$

$$= \cancel{10{,}4436}\ 1{,}364512$$

$$\Rightarrow (0{,}43,\ 3{,}15)$$

c) The estimators for μ and β will not change including this additional observation ~~that~~ since it lies on the current regression curve and thus is $E(Y|x=15.4)$.

3a)

$T_{\cdot j} = n_j \overline{Y}_{\cdot j}$

$T_{\cdot 1} = 14 \cdot 14.5 = 203$

$T_{\cdot 2} = 211.4$

$T_{\cdot 3} = 248$

$T_{\cdot\cdot} = 662.4$

$C = \frac{T_{\cdot\cdot}^2}{\sum_j n_j} = 9972.131$

$SSTR = \sum \frac{T_{\cdot j}^2}{n_j} - C = 7.509$

$SSE = \sum (n_j - 1) S_j^2 = 8.27$

$SSTOT = 15.78$

Source	df	SS	MS	F
Treatment	$k-1$ 2	7.7509	3.75	18.6
Error	$44-3=41$ ~~43~~	8.27	0.20	
Total	43	15.78		

$F = \frac{MSTR}{MSE}$

$Obs. F = 18.6 > F_{.95,2,41} = 3.22$

$\Rightarrow$ we reject $H_0 : \mu_1 = \mu_2 = \mu_3$

b) . H_0: $\mu_{medium} = \frac{1}{2}\mu_{low} + \frac{1}{2}\mu_{high}$

H_1: $\mu_{medium} \neq \frac{1}{2}\mu_{low} + \frac{1}{2}\mu_{high}$

$$C = \frac{1}{2}\mu_{low} + \frac{1}{2}\mu_{high} - \mu_{medium}$$

$$SS_c = \frac{(\sum c_j \bar{Y}_{\cdot j})^2}{\sum \frac{c_j^2}{n_j}} = 0.09$$

$$F_{.95,1,41} = 4.07$$

$\Rightarrow$ we cannot reject H_0.

Oppgave 4. Fysisk eller digital undervisning?

a) Forutsetninger: $D_1, D_2, \dots, D_{16}$ uavh. og $N(\mu_D, \sigma_D^2)$

$H_0: \mu_D = 0$ mot $H_1: \mu_D \neq 0$

Testobs:

$$T = \frac{\bar{D}}{S_D/\sqrt{16}} = \frac{4\bar{D}}{S_D} \sim t\text{-fordelt under } H_0.$$

Her er $\bar{D} = \frac{80}{16} = 5.0$

$$S_D^2 = \frac{1}{15}\sum_{i=1}^{16}(D_i - \bar{D})^2 = \frac{1}{15}\left(\sum_{i=1}^{16} D_i^2 - 16\bar{D}^2\right)$$

$$= \frac{1}{15}\left[1604 - 16 \cdot 5^2\right] = 80.27$$

$$S_D = \sqrt{} = 8.96$$

$$T = \frac{4 \cdot 5.0}{8.96} = 2.23$$

Sammenlign med $\pm t_{15, 0.025} = \pm 2.131$.

dvs Forkast H_0 på 5% nivå.

b) Sett her $p = P(X_i > Y_i)$; $i = 1, 2, \ldots, 16$

Det testes $H_0 : p = \frac{1}{2}$.

Testobservator er

$$U = \text{antall par } (X_i, Y_i) \text{ der } X_i > Y_i$$

De n parene $(X_1, Y_1), \ldots, (X_n, Y_n)$ antas uavhengige. Da er

$$U \sim bin(16, \tfrac{1}{2}) \text{ under } H_0$$

og vi får p-verdien

$$2 \min\{P(U \leq \text{observert verdi}), P(U \geq \text{obs. verdi})$$

Her observeres $U = 11$ og p-verdien blir

$$2 \cdot P(U \geq 11) = 2(1 - P(U \leq 10)$$

$$= 2 \cdot (1 - 0.895) = 2 \cdot 0.105 = \underline{\underline{0.21}}$$

bruk tabell med $n = 16$ $p = 0.5$

Vi forkaster altså <u>ikke</u> H_0 med tegn-testen

c) Her antas igjen at parene (X_i, Y_i) er uavhengige. Man baserer seg igjen på $D_1, D_2, \ldots, D_n$ og antar at disse har en symmetrisk fordeling.

Testobservatoren er sum av rangen for de positive d_i-ene:

Ordne dem først:

2	2	2	4.5	4.5	6	7	8.5	8.5	10	11
−2	2	2	−3	−3	−4	6	7	7	8	11

12	13	14	15	16
12	13	−14	17	21

~~$W = 2+2+7+8+8+8+11+12+13+17+21 = 105$~~

$$W_+ = 2+2+7+8.5+8.5+10+11+12+13+15+16$$
$$= 105$$

$$W_- = 2+4.5+4.5+6+14 = 31$$

Sum: $136 = \frac{16 \cdot 17}{2}$ #

Under H_0 er $E(W_+) = \frac{n(n+1)}{4} = \frac{136}{2} = 68$

$$Var(W_+) = \frac{n(n+1)(2n+1)}{24} = 374$$

Dermed bli

$$P(W_+ \geq 105) = 1 - P(W_+ \leq 104)$$

$$= 1 - \Phi\left(\frac{104.5 - 68}{\sqrt{374}}\right)$$

$$\cancel{= 1 - \Phi(1.370282)}$$

$$= 1 - \Phi(1.89)$$

$$= 1 - 0.9706 = 0.0294$$

Multiply by 2: <u>0.0588</u> ← to-sidig p-verdi.

d) Bruker tabell for bin(16, 0.5):

D_a er

$P(V \le 3) = 0.011 \le 0.025$

$P(V \ge 13) = 1 - P(V \le 12) = 1 - 0.989 = 0.011 \le 0.025$

Videre er

$P(V \le 4) = 0.038 > 0.025$

$P(V \ge 12) = 1 - 0.962 = 0.038 > 0.025$

Altså vil en "5%" test forkaste hvis $V \le 3$, $V \ge 13$.

$$P(\text{Type I-feil}) = P(\text{forkaste } H_0 \text{ hvis } p = \tfrac{1}{2})$$
$$= P(V \le 3) + P(V \ge 13)$$
$$= 0.011 + 0.011 = \underline{\underline{0.022}}$$

$D_1, D_2, \ldots, D_{16} \sim N(\mu_D, 10^2)$, $\bar{D} \sim N(\mu_D, \frac{10^2}{16})$
$= N(\mu_D, 2.5^2)$

Hvis $\mu_D = 0$:

$$P(\bar{D} \le -5.725) = P\left(\frac{\bar{D}}{2.5} \le -\frac{5.725}{2.5}\right)$$

$$= \Phi(-2.29) = 0.011$$

Likeså, $P(\bar{D} \ge 5.725) = 0.011$

så $P(\text{type I-feil}) = 0.011 + 0.011 = \underline{0.022}$

Anta $\mu_D = \cancel{\ldots}\ 4.0$

<u>Tegntesten</u>: Nå er $V \sim \text{bin}(16, p)$

der

$$p = P(X_i > Y_i) = P(D_i > 0)$$

$$= \cancel{P\left(\frac{D_i - 5.25}{10}\right)}$$

$$= P\left(\frac{D_i - 4}{10} > \frac{0 - 4}{10}\right)$$

$$= \Phi(0.400) \approx 0.6554$$

Da blir P(type II-feil) ved tegntesten

$$P(4 \le V \le 12) = P(V \le 12) - P(V \le 3)$$

$$= 0.754 - 0 = \cancel{0.6554}$$

tabell $n=16$, $p=0.7$: 0.754

$p=0.6$: 0.935

gj.sn. 0.85 ca

ca. 0.85 gjennomsnitt

Exact: 0.8563

<u>t-testen</u>: Nå er $\bar{D} \sim N(4, 2.5)$

så

$$P(\text{type II-feil}) = P(-5.725 < \bar{D} < 5.725)$$

$$= P\left(\frac{-5.725 - 4}{2.5} < \frac{\bar{D} - 4}{2.5} < \frac{5.725 - 4}{2.5}\right)$$

$$= P(\cancel{-4.39} < \text{"}Z\text{"} < 0.19)$$

$$P(-3.89 < \text{"}Z\text{"} < 0.69) = \Phi(0.69) = 0.7549$$

$$= \Phi(0{,}19) = \underline{\underline{0{,}5753}}$$

"t-testen" er altså "sterkere" enn tegn-testen. Sistnevnte bygger imidlertid på mye svakere antagelser.

Merk at det ble antatt at σ_D er kjent, dvs. at det ved bruk av t-test kommer inn en ekstra usikkerhet i forbindelse med estimering av σ_D.