Problem 1

$X_1, \dots, X_n \sim N(\mu, 0.03^2)$

$H_0: \mu = 0.2$ $\qquad \alpha = 0.05$,

$H_1: \mu > 0.2$

a) $n = 16 \quad \Rightarrow \quad \bar{X} \sim N(\mu, \frac{0.03^2}{n})$

$$\sqrt{n} \, \frac{\bar{X} - \mu}{0.03} \sim N(0,1)$$

We use a normal test with testobservator

$$z = \sqrt{16} \, \frac{0.22 - 0.2}{0.03} = 2.11$$

$z_{0.95} = 1.64 \Rightarrow$ the null hypothesis is rejected

b) We know $X_i \sim N(\mu^*, \sigma^2)$ $\qquad \mu^* = 0.22$

$$P(\sqrt{n} \, \frac{\bar{X} - \mu_0}{\sigma} \geq z_\alpha) = P(\sqrt{n} \, \frac{\bar{X} - \mu^* + \mu^* - \mu_0}{\sigma} \geq z_\alpha)$$

$$= P(\sqrt{n} \, \frac{\bar{X} - \mu^*}{\sigma} \geq z_\alpha - \sqrt{n} \, \frac{\mu^* - \mu_0}{\sigma})$$

$$= P(Z \geq z_\alpha - \sqrt{n} \, \frac{\mu^* - \mu_0}{\sigma})$$

$$= 1 - \Phi\left(z_\alpha - \sqrt{n}\,\frac{\mu^* - \mu_0}{\sigma}\right) =$$

$$= \Phi\left(\sqrt{n}\,\frac{\mu^* - \mu_0}{\sigma} - z_\alpha\right)$$

$$= \Phi(0.463) = \underline{\underline{0.678}}$$

The probability to reject the null hypothesis with $n = 10$ and $\mu = 0.22$ is 67.8%

c) We are interested in

$$\Phi\left(\sqrt{n}\,\frac{\mu^* - \mu_0}{\sigma} - z_\alpha\right) \geq 0.8$$

or

$$\Phi\left(\frac{0.22 - 0.2}{0.03}\sqrt{n} - 1.645\right) \geq 0.8$$

$$\Phi(0.6\bar{6}\sqrt{n} - 1.645) \geq 0.8$$

From the table we find that

$$0.6\bar{6}\sqrt{n} - 1.645 \geq 0.85 \Rightarrow \underline{\underline{n \geq 15}}$$

d)

	Blue	Yellow	Red	Green
Expected	$\frac{1}{4} 112$	$\frac{1}{4} \cdot 112$	$\frac{1}{4} \cdot 112$	$\frac{1}{4} \cdot 112$
Observed	31	29	23	29

$\Rightarrow$ Goodness of fit test (parameters known)

$$\chi^2 = \frac{(28-31)^2}{28} + \frac{(28-29)^2}{28} + \frac{(28-23)^2}{28} + \frac{(28-29)^2}{28}$$

$$= 1,29$$

$$\chi^2_{0.05,3} = 7.815 \Rightarrow H_0 \text{ is not rejected}$$

$\Rightarrow$ There is no indication that the colour of the box influences its sales.

Problem 2

a) The change would be $4 \cdot \hat{\beta}_1 = 4 \cdot (-0.013) = -0.0521$

In year 0 the winning time would be

$\hat{\beta}_0 = 36.78$ s.

b) We would like to test

$H_0: \beta_1 = 0$ vs $H_0: \beta_1 < 0$

The test statistic is

$$T = \frac{\hat{\beta}_1}{\sqrt{\frac{\hat{\sigma}^2}{\sum(x_i - \bar{x})^2}}}$$

$$= \frac{-0.013}{\sqrt{\frac{0.036}{9120}}} = -6.54$$

$$t_{0.05,\underset{19}{\underset{\uparrow}{n}}-2} = t_{0.05,17} = -1.740$$

$\Rightarrow$ We reject H_0. The down ward trend seems to be real.

Problem 3:

	7	8	9	10	11	12	14
Group A	114	203	217	254	256	284	296
Group B	4	7	21	25	48	71	294
	1	2	3	4	5	6	13

outliers

$$W_B = 1 + 2 + 3 + 4 + 5 + 6 + 13 = 34$$

$$p\text{-value} = 2\min\left(P(W_B \leq 34),\ P(W_B \geq 34)\right)$$

$$\frac{n_B(n_B+1)}{2} = \frac{7 \cdot 8}{2} = 28$$

$$P(W_B \leq 34) = P(W_B - 28 \leq 6)$$

$$= P(U_1 \leq 6) \overset{\text{Tabell}}{=} 0.009$$

$$P(W_B \geq 34) = P(U_1 \geq 6) = 1 - P(U_1 \leq 5)$$

$$= 1 - 0.006 = 0.994$$

$$\Rightarrow \quad p\text{-value} = 2 \cdot 0.009 = 0.018$$

With a significance level of 5% we reject the null hypothesis that both forms of treating the dog are equal

Petting seems to be preferred.

a) Variansanalysetabell

Kilder	DF	SS	MS	F
Kategori	4	4.6	1.15	1.15/0.501 = 2.3
Feil	45	22.9	0.501	
Total	49	27.5		

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$ H_1: Minst 2 forventingar er forskjellige

$F_{obs} = 2.3 < 2.79 = f_{0.05,4,50} < f_{0.05,4,45}$

$\Rightarrow$ Ingen grunn til å forkaste H_0 på 5% nivå.

b) $C = \mu_4 - \frac{1}{4}(\mu_1 + \mu_2 + \mu_3 + \mu_5)$

$H_0: \mu_4 = \frac{1}{4}(\mu_1 + \mu_2 + \mu_3 + \mu_5)$ $H_1: \mu_4 \neq \frac{\mu_1 + \mu_2 + \mu_3 + \mu_5}{4}$

$\hat{C} = \bar{Y}_{.4} - \frac{\bar{Y}_{.1} + \bar{Y}_{.2} + \bar{Y}_{.3} + \bar{Y}_{.5}}{4}$. $SS_C = \frac{\hat{C}^2}{\frac{1}{10}\left(1 + \frac{4}{16}\right)}$

$\Rightarrow F = \frac{\hat{C}^2}{MSE \cdot \frac{5}{40}} \sim F_{1,45}$. $F_{obs} = \frac{(0.625)^2 \cdot 40}{0.501 \cdot 5} = 6.24 > 4.08$

$= f_{0.05,1,40} > f_{0.05,1,45} \Rightarrow$ forkast H_0 på nivå 0.05

c)

$$SSTOT = \sum_{j=1}^{5} \sum_{i=1}^{10} (y_{ij} - \bar{y}_{..})^2 = \sum_{j=1}^{5} \sum_{i=1}^{10} (y_{ij}^2 - 2 y_{ij} \bar{y}_{..} + \bar{y}_{..}^2)$$

$$= \sum_{j=1}^{5} \sum_{i=1}^{10} y_{ij}^2 - 2 \sum_{j=1}^{5} y_{.j} \cdot \bar{y}_{..} + 50 \bar{y}_{..}^2$$

$$= \sum_{j=1}^{5} \sum_{i=1}^{10} y_{ij}^2 - 2 y_{..} \bar{y}_{..} + 50 \bar{y}_{..}^2 = \sum_{j=1}^{5} \sum_{i=1}^{10} y_{ij}^2 - 2 \frac{y_{..}^2}{50} + 50 \cdot \frac{y_{..}^2}{50^2}$$

$$= \sum_{j=1}^{5} \sum_{i=1}^{10} y_{ij}^2 - \frac{y_{..}^2}{50} .$$

Sidan $y_{..} = 10 \sum_{j=1}^{5} \bar{y}_{.j}$ har vi vist resultatet.

$$\sum_{j=1}^{5} \sum_{i=1}^{10} y_{ij}^2 = SSTOT + \frac{y_{..}^2}{50} = 27,5 + \frac{25^2}{50} = 40$$