Stochastic population modeling

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Density regulation

Density regulation is most appropriately introduced by assuming that the expected contributions to the next generation, $\lambda = \mathrm{E}w$, depends on the population size, $\lambda = \lambda(N)$.

Correspondingly, the deterministic growth rate is then $r(N) = \ln(N)$.

Define the carrying capasity K as the stable equilibrium where $s(K) = 0$; $s(N) = \mathrm{E}(\ln \Lambda | N)$

Example: Independent pop. sizes

Ch. 1: Distribution of $\Delta \ln N$, given N, does not depend on N.

Now assume that $\ln N_t \sim N(\mu,\sigma^2)$

Then:

$$s(N)=E(\Delta \ln N|N)=E(\ln(N+\Delta N)|N)-\ln(N)=\mu-\ln(N)$$

And:

$$\sigma_e^2=Var(\Delta \ln N|N)=\sigma^2$$

2.2 Return time to equilibrium and strength of density regulation

Define *relative deviation* $\varepsilon=(N-K)/K$

Return time to equilibrium:

T_R time required for the deviation to reach a fraction 1/e of its original value.

$T_R = -1/(K\lambda'(K))$

Strength of density regulation:

$$\gamma = \frac{1}{T_R} = -K\lambda'(K)$$

2.4 log-linear model / Gompertz type of density regulation

$$\lambda(N) \approx \lambda(K) + \frac{d\lambda}{dX}\Big|_{N=K} (X - \ln K) = \lambda(K) + \gamma(\ln K - \ln N)$$

$$\frac{\Delta N}{N} = \lambda(N) - 1 = \gamma(\ln K - \ln N) = \gamma \ln K \left(1 - \frac{\ln N}{\ln K}\right)$$

or $$\Delta N = \gamma N \ln K \left(1 - \frac{\ln N}{\ln K}\right)$$

on log-scale $$\Delta X = \gamma k \left(1 - \frac{x}{k}\right), \quad k = \ln K$$

Linear in *X* -> log-linear model

2.5 Theta-logistic model

$$r(N) = r_0\left[1-\left(\frac{N}{K}\right)^{\theta}\right]$$

$$r(N) = r_1\left[1-\left(\frac{N^{\theta}-1}{K^{\theta}-1}\right)\right], \quad \theta \neq 0$$

$$\theta = 0 \quad \Rightarrow \quad r(N) = r_1\left(1-\frac{\ln N}{\ln K}\right)$$

Fig. 2.1

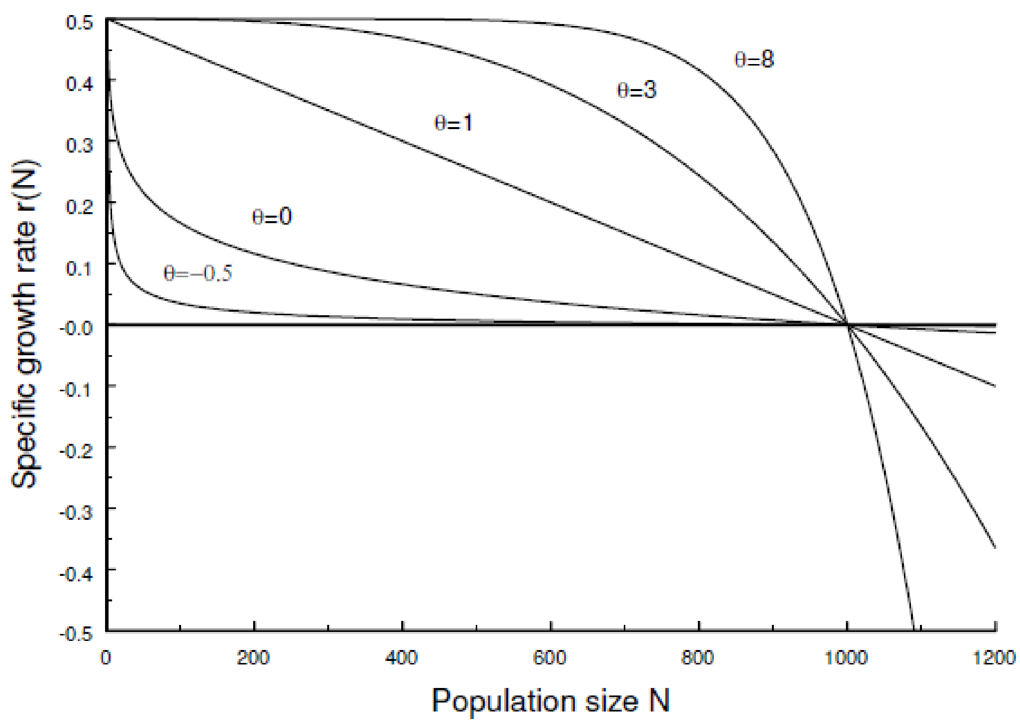


Figure 2.1: The deterministic growth rate $r(N)$ as a function of N for different values of θ in the thetalogistic model. The other parameters are $r_1 = 0.5$ and $K = 1000$.

2.6 Stochasticity and density-regulation

The most common way of formulating a stochastic model is to assume that the parameter expressing the population growth rate at small densities fluctuates in time. But this assumption alone is not enough to uniquely define a model.

Overall goal = formulate a model that is realistic for the population we study.

Stochasticity and density-regulation

Deterministic model:

$$\Delta \ln N = r - \frac{1}{2}\sigma_e^2 - \frac{1}{2N}\sigma_d^2 - g(N)$$

Replacing *r* by a temporally fluctuating *r(t)*, we obtain the stochastic model

$$S_t = \Delta \ln N = r(t) - \frac{1}{2}\sigma_e^2 - \frac{1}{2N}\sigma_d^2 - g(N)$$

Notice: No stochasticity in the density regulating term.

Stochasticity and density-regulation

Hence, the variance in $\Delta \ln N$, given the population size the previous year, is:

$$Var\left(S_t \mid N\right) = Var\left(r\left(t\right)\right) \approx Var\left(\Delta N / N\right) = \sigma_e^2 + \sigma_d^2 / N$$

For the discrete theta-logistic model:

$$S_t = \Delta \ln N = r_1\left(t\right) - \frac{1}{2}\sigma_e^2 - \frac{1}{2N}\sigma_d^2 - \bar{r}_1 \frac{N^\theta - 1}{K^\theta - 1}$$

2.7 Density dependence in dem. and env. variances

If $E\left(\frac{\Delta N}{N} \mid N\right)$ depend on *N*, so may σ_e^2 and σ_d^2

Ex: Let *J*=indicator of survival for an individual in an environment z with expected value $p(z)$, then $E(J) = E\left[p(z)\right]$

and

$$Var(J) = E\left[Var(J \mid z)\right] + Var\left[E(J \mid z)\right] = E\left[p(z)(1 - p(z))\right] + Var\left[p(z)\right]$$

If $E(J)$ depend on N, so will $Var(J)$

Fig 2.2

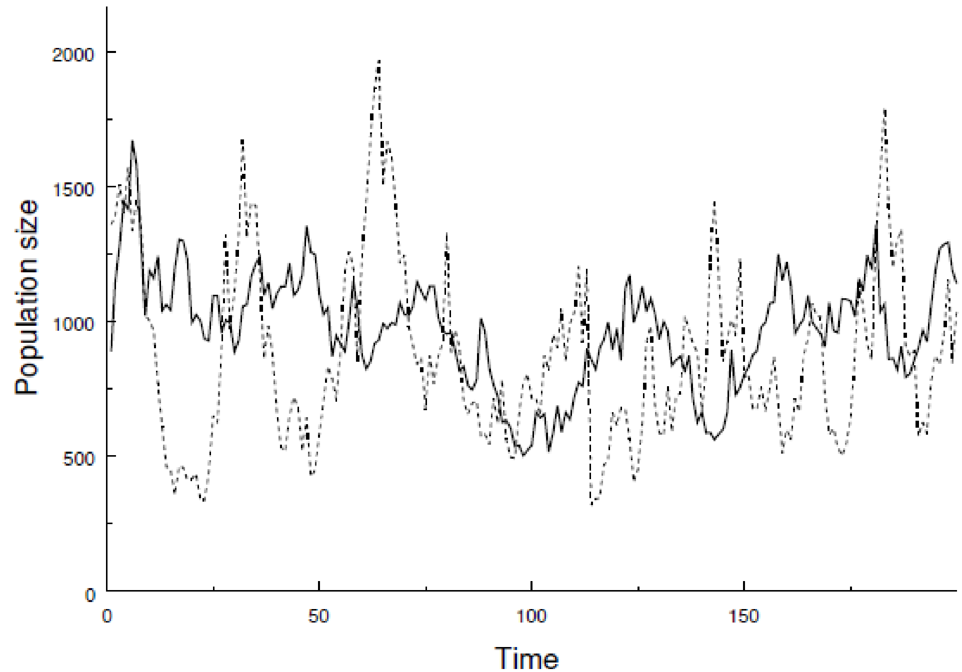
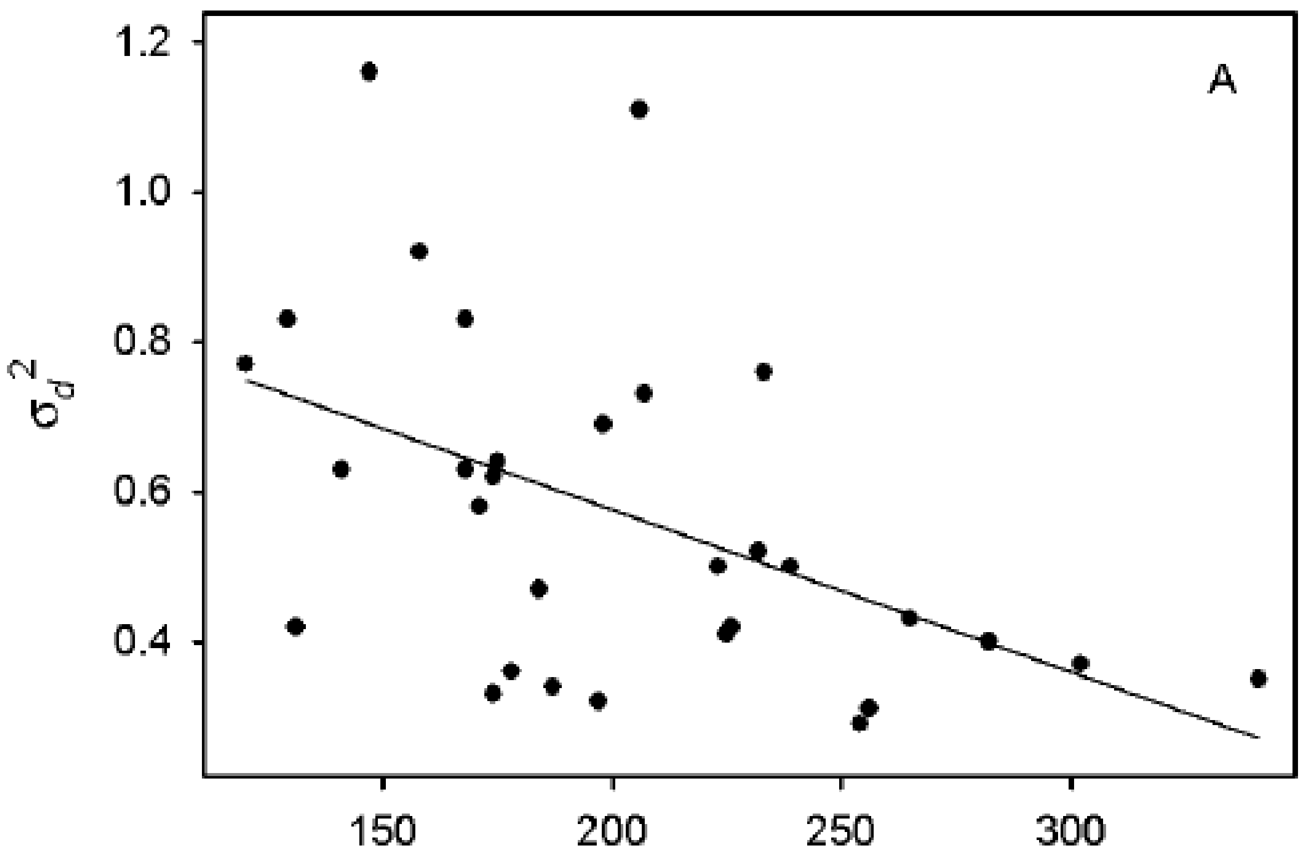


Figure 2.2: Simulations of the multiplicative process with $\sigma^2 = 0.01$ (solid line) and 0.04 (dotted line). The other parameters are $K = 1000$ and $\gamma = r = 0.1$, corresponding to a return time to equilibrium $T_R = 1/\gamma = 10$.

Fig 2.4



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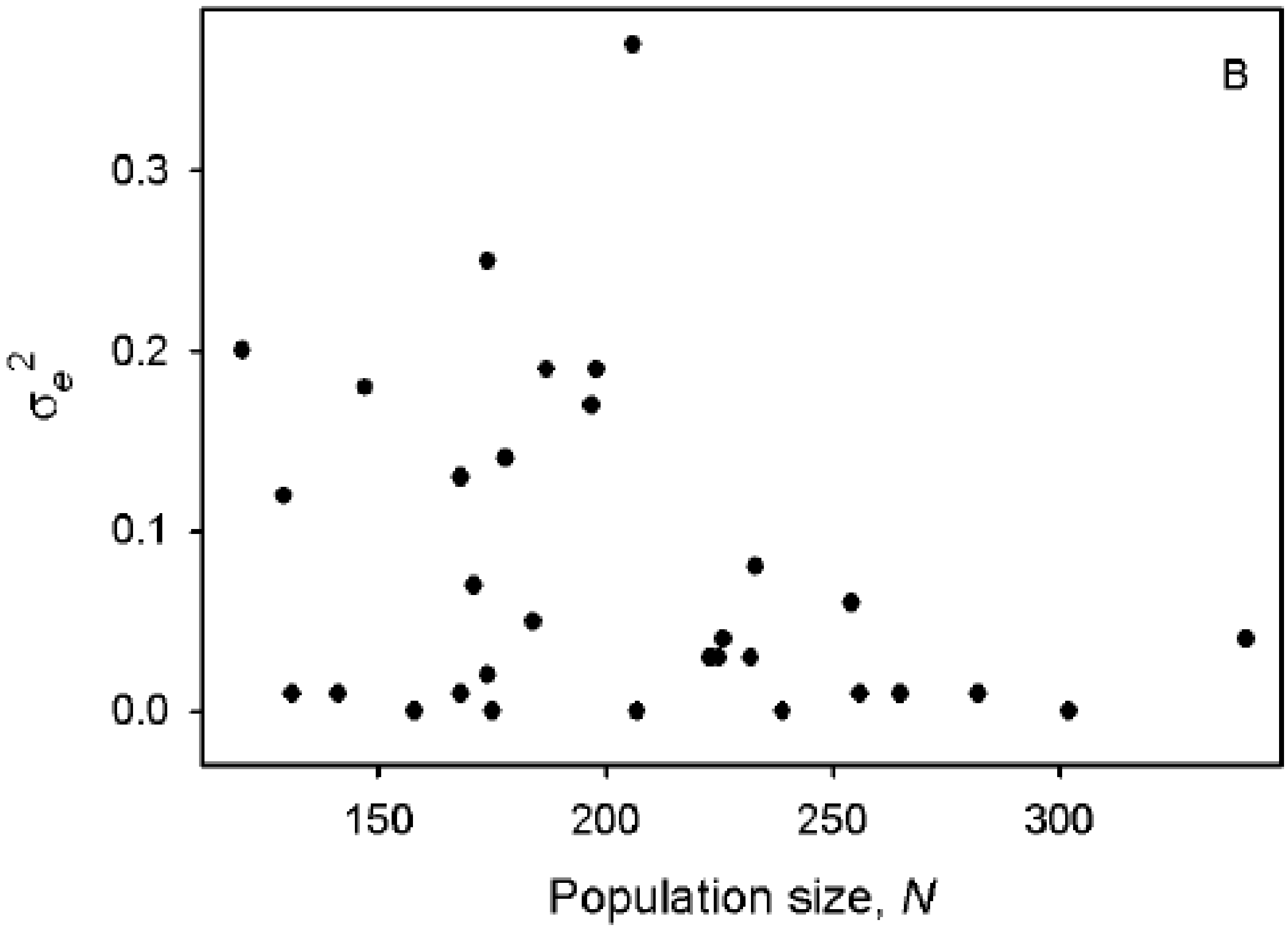
B
0.3
0.2
0.1
0.0
σ_e^2
150
200
250
300
Population size, N