Solution of assignment 3, ST2304

Problem 1

1.
```
> anova(helimod)
Analysis of Variance Table

Response: flighttime
          Df Sum Sq Mean Sq F value    Pr(>F)
size       1  5.558   5.558  3.6845 0.0700664 .
wing       2 91.583  45.791 30.3532 1.213e-06 ***
clip       1 23.384  23.384 15.5003 0.0008847 ***
Residuals 19 28.664   1.509
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```
The model can be written as

$$\text{flighttime} = \mu + \alpha_{\text{size}} + \beta_{\text{wing}} + \gamma_{\text{clip}} + \epsilon, \quad \epsilon \sim N(0, \sigma^2), \tag{1}$$

where the factors size, wing and clip are factors with levels

$$\begin{aligned} \text{size} &= \text{large}, \text{small} \\ \text{wing} &= \text{control}, \text{up}, \text{down} \\ \text{clip} &= \text{no}, \text{yes} \end{aligned}$$

The model has $1 + 2 + 3 + 2$ parameters in addition to σ^2. Not all these parameters can be estimated, therefore the constraint that the effect sizes in the control groups are zero is imposed, i.e. $\alpha_{\text{large}} = 0$, $\beta_{\text{control}} = 0$ and $\gamma_{\text{no}} = 0$. The model can be rewritten as the multiple regression

$$\begin{aligned} \texttt{flighttime} = \mu &+ \alpha_{\text{small}} x_{\text{small}} \\ &+ \beta_{\text{up}} x_{\text{up}} + \beta_{\text{down}} x_{\text{down}} \\ &+ \gamma_{\text{yes}} x_{\text{yes}} \\ &+ \epsilon \end{aligned}$$

where the x's are dummy variables indicating the level of each factor. For each factor the number of terms are equal to the number of levels of each factor minus 1.

For a balanced design such as this, the total variantion in flighttime decomposes into sum of squares for each factor plus the residuals sum of squares. Based on the sum of squares size , wing and clip thus explains 3.7, 61 and 15.9% of the total variation, respectively. Summing up, this gives a proportion of 80.79% of the total variation (which we can also find in the "Multiple R-squared" in the second last line in summary())

Clearly how the wing is folded contribute most to the variation in flighttime(61%).

2. Both wing and clip show a significant effect on flighttime, as there p-value are smaller then significant level of α=0.05

3. We see that size is not significant at the level of α=0.05, as the p-value is larger than α.

```
> anova(helimod)
Analysis of Variance Table

Response: flighttime
          Df Sum Sq Mean Sq F value    Pr(>F)
wing       2 91.583  45.791  26.761 2.219e-06 ***
clip       1 23.384  23.384  13.666  0.001428 **
Residuals 20 34.222   1.711
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

4. Since we have a balanced design, the sum of squares for wing and clip does not change, and the residual sum of squares (SS_E) increases from 28.664 to 34.222 by an amount equal the sum of squares for size. The p-value for both clip and wing changes but the changes are small, again as a result of the balanced design.

5. > summary(helimod)

```
Call:
lm(formula = flighttime ~ size + wing + clip)

Residuals:
     Min       1Q   Median       3Q      Max
-2.29458 -0.59240  0.09708  0.77948  2.13583

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  10.8308     0.5606  19.319 5.97e-14 ***
sizesmall    -0.9625     0.5014  -1.919 0.070066 .
wingdown     -3.8925     0.6141  -6.338 4.41e-06 ***
wingup       -4.3562     0.6141  -7.093 9.53e-07 ***
clipyes      -1.9742     0.5014  -3.937 0.000885 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.228 on 19 degrees of freedom
Multiple R-squared: 0.8079,Adjusted R-squared: 0.7674
F-statistic: 19.97 on 4 and 19 DF,  p-value: 1.356e-06
```

The estimated effect of small size on flighttimerelative to large size is -0.9625 seconds. Which means that having a large size have a longer flighttime. However, the difference seems to be small.

6. We see that the estimated effect of wingdownon flighttimerelative to the effect of wingcontrolis -3.8925 seconds and the estimated effect of wingupon flighttimerelative to the effect of wingcontrolis -4.3562 seconds. Thus, folding the wings up decreases the flighttime the most and folding the wings down decreases the flighttime, but not as much as folding it up. To test if there is a difference between the up and down treatments we may setting "down" to be the control-group

```
wing <- relevel(wing,"down")
```

and refit the model.

7. Attaching a paper clip to the helicopter decreases the flighttime-1.9742 seconds in relation to not attaching a paper clip.

8. The effect on size is not significant on the flighttime.

9. If we remove size the model, the paramter estimates become

```
> summary(helimod)

Call:
lm(formula = flighttime ~ wing + clip)

Residuals:
     Min       1Q   Median       3Q      Max
-2.57542 -0.53208  0.09063  0.68010  2.61708

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   10.350      0.534  19.380 1.97e-14 ***
wingdown      -3.893      0.654  -5.951 8.06e-06 ***
wingup        -4.356      0.654  -6.660 1.75e-06 ***
clipyes       -1.974      0.534  -3.697  0.00143 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.308 on 20 degrees of freedom
Multiple R-squared: 0.7706,Adjusted R-squared: 0.7362
F-statistic:  22.4 on 3 and 20 DF,  p-value: 1.328e-06
```

   Because the design is balanced, the point estimates of the parameter do not change. Note also that the change in the standard errors and associated t-test is also very small.

10. The assumption of additivity might be somewhat unreasonable in that the predicted values by the model in theory may take negative values. A more realistic model could perhaps be built based on what is known from physics about terminal velocity of free falling objects.

    R-code

```
#download dataset
heli <- read.csv("/home/anna/Documents/st2304/helicopterdata.csv")
attach(heli)

#three-way anova
helimod <- lm(flighttime ~ size + wing + clip)

#anova table for the model
anova(helimod)

#fitted reduced model
```

```
helimod <- lm(flighttime ~  wing + clip)

#summary of the full model
helimod <- lm(flighttime ~ size + wing + clip)
summary(helimod)

#re-leveling the factors (choosing another factor as "reference")
wing <- relevel(wing,"down")
```