Statistical Inference: Uncertainty About One Parameter

Recap of Last Week

We tossed a beach ball around

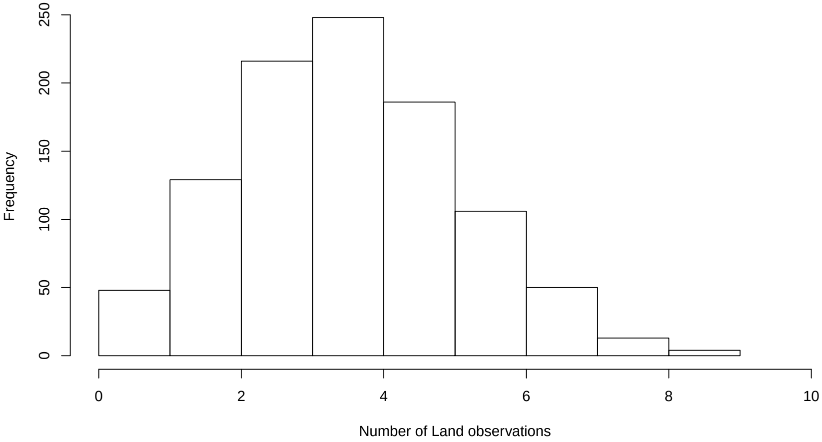
We saw land 6 times and sea 7

The second time we saw land 7 times and sea 6

We want to estimate the proportion of land

Recap of Last Week

We saw that there would be variation when we replicate the experiment



Recap of Last Week

We can estimate the proportion by

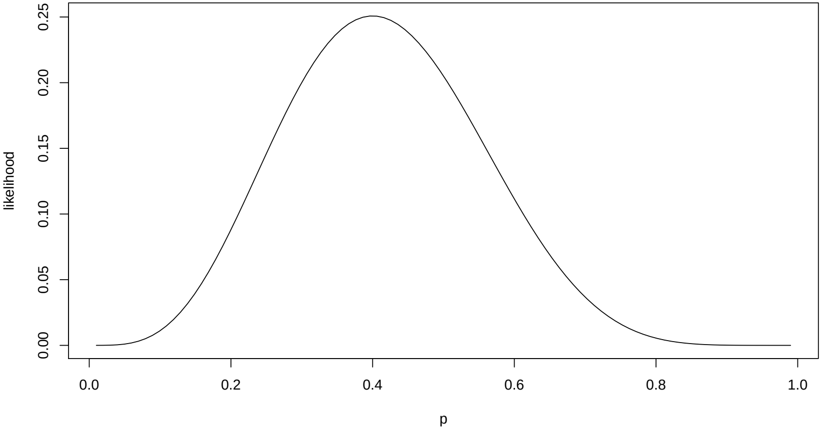
- building a model
- finding the parameters that are model likely to give the data

This is the *maximum likelihood estimate*

- maximise Pr(Data|parameters) with respect to the parameters

Recap of Last Week

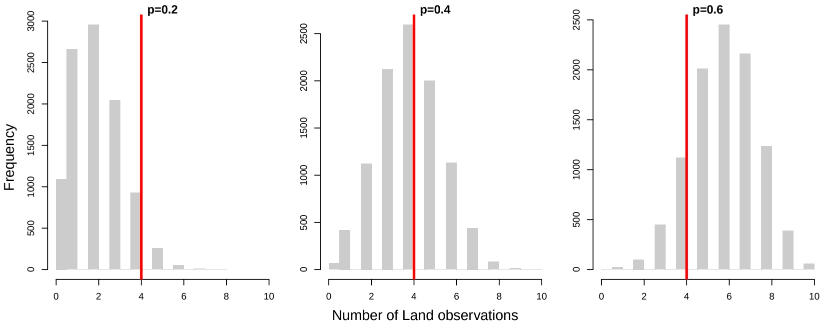
For this problem we can maximise the likelihood analytically



This week

How good is our estimate?

We saw last week that different values of p can give the same data



Outline

Repeated sampling of data

Summarising the variation in the resamples - confidence intervals

What is a confidence interval?

Asymptotics: approximations when the numbers are big

Standard errors

The Question

Because different samples give different estimates, we want to quantify this - suggest plausible values

What summaries could we use?

(What summaries do we use for simple statistics?)

Simulating the Sampling Distribution

From our data, we have our estimate of p (which we call $\hat{p}$)

If this is the true value, what values are we likely to estimate?

What to do

Simulate the data. For each simulation calculate $\hat{p}$, the maxmimum likelihood estimate of p.

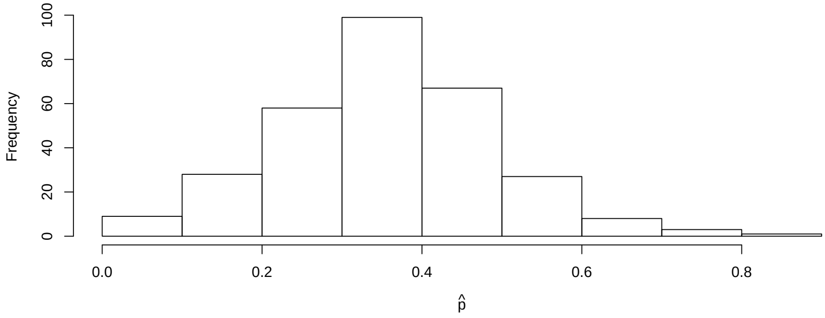
Look at the histogram of the distribution

- code on next 2 slides

Simulations of the sampling distribution

We know that the MLE for p is r/N, e.g. Land/(Land + Sea), so we can calculate it from the simulations: the web link is "https://www.math.ntnu.no/emner/ST2304/2020v/Week02/Week2Functions.R"

```r
source("https://www.math.ntnu.no/emner/ST2304/2020v/Week02/
sim <- simGlobe(probability=0.4, NTrials=10, nSims = 300)
hist(mleGlobe(sim["Land",], NTrials = 10),
     xlab=expression(hat(p)), main="")
```



How can we summarise the distribution?

Can we give a range of probable values?

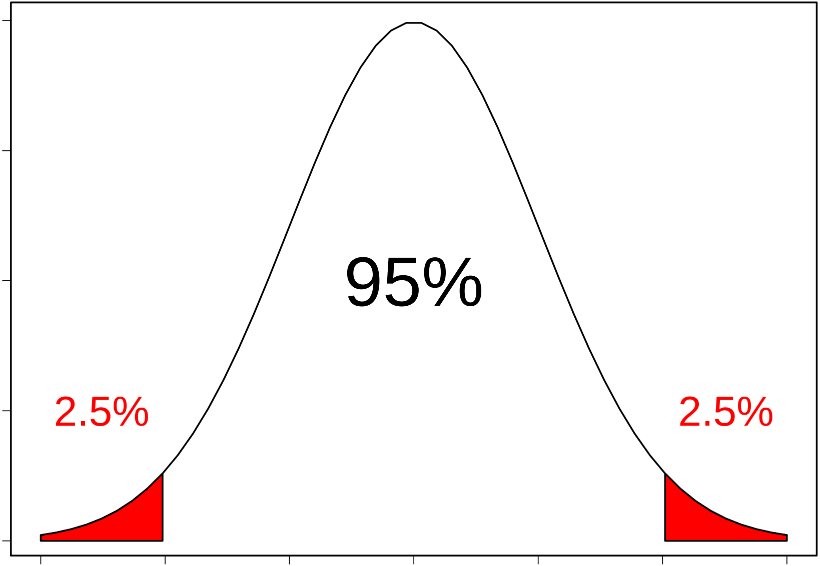
Confidence Intervals

We can give an interval within which we think we would see the sample statistic

- the confidence interval
- usually use 95%

Confidence Intervals

For continuous data the 95% confidence interval is constructed like this



Confidence Intervals

Your task: try to calculate an approximate 95% confidence interval for your data

Your task: follow the "Constructing a Confidence interval" section

(for discrete data it is a bit more difficult to get an exact interval)

Confidence Intervals and Quantiles I

There are a few ways to calculate confidence intervals. One way is to sort the numbers from lowest to highest

```
SimDist1k <- simGlobe(probability=0.4,
                      NTrials=1e3,
                      nSims = 1e3)["Land",]
sort(SimDist1k)[1:10]
```

```
##  [1] 349 351 354 359 362 362 363 364 365 365
```

and then take the values that are 2.5% of the way from the bottom, and 2.5% of the way from the top:

```
sort(SimDist1k)[c(0.025*length(SimDist1k),
                  0.975*length(SimDist1k))]
```

```
## [1] 371 432
```

Confidence Intervals and Quantiles II

The values 2.5% of the way from the bottom, and 2.5% of the way from the top are called **quantiles**, specifically the 2.5% and 97.5% quantiles.

A x% quantile is a values of a distribution with x% of the distribution less than it

- a median is the 50% quantile
- the 25% and 75% quantiles are called quartiles (they plus the median split the data into 4 quarters)

So, we can just need the 2.5% and 97.5% quantiles. There is a function in R to do this:

```
quantile(SimDist1k, c(0.025, 0.975))
```

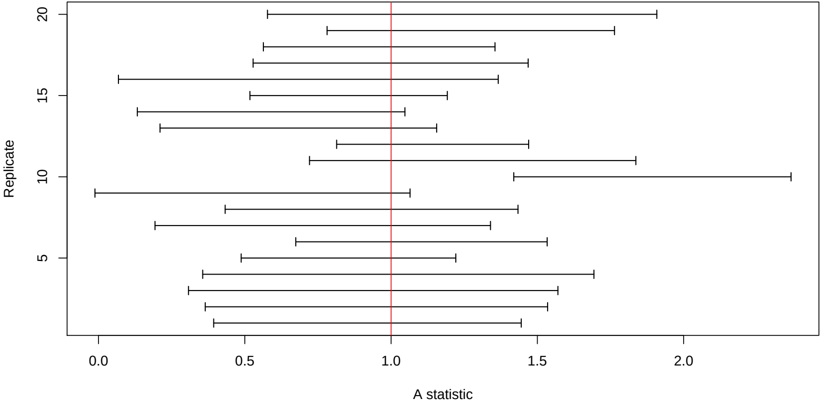
```
##  2.5% 97.5%
##   371   432
```

Confidence Intervals and Quantiles: Your tasks

Your task: follow the "Confidence Intervals and Quantiles" section

OK, so what, exactly, is a confidence interval?

A confidence interval is an interval that will contain a population parameter a specified proportion of the time.



i.e. if we repeatedly sample the same population, 95% of confidence intervals will include the “true” parameter Your task: follow the "What, exactly, is a confidence interval?" section

Confidence Intervals with more data

Now imagine that rather than 10 trials, you have 1000. As before, you see 40% of the observations are land (i.e. 400 out of 1000)

```
# 1e3 = 1x10^3 = 1000
sim <- simGlobe(probability=0.4, NTrials=1e3, nSims = 3)
```

Try to find a 95% confidence interval for this

Basically, we want to remove the outer 2.5% of values, and see what is left.

Confidence Intervals with more data

Your task: follow the "Confidence Interval for a Binomial with differnet Ns" section

What are the differences in the confidence intervals?

- in their size
- in how well they cover 95% of the sampling distribution

Asymptotic Confidence Intervals

In statistics, large numbers usually make things much nicer: there are a lot of asymptotic results (i.e. approximations that work well when there is a lot of data).

One of these is the that most sampling distributions of statistics look like normal distributions, with enough data.

So, if we can construct a normal distribution's CI, we can make an approximation.

Normal Confidence Intervals

We can calculate a normal confidence interval like this:

```
c(qnorm(0.025, mu, sigma), qnorm(0.975, mu, sigma))
```

The parameters are the mean and standard deviation, e.g.

```
## [1] -5.839856  9.839856
```

Normal Approximations

If we know the mean and standard deviation of the sampling distribution, then we can use a normal approximation.

- the standard deviation of the sampling distribution is called the standard error

Normal Approximation for the Binomial

We can use the MLE, $\hat{p}$ as the mean of the normal

The standard error for the binomial distribution is

$$\frac{p(1-p)}{\sqrt{n}}$$

Approximatioms: Your tasks

Your task: follow the "Approximations" section

How well are we doing?

With small N, we cannot usually get a perfect 95% confidence interval, because the possible esitmates are discrete $(0/N, 1/N, ..., N/N)$, so our interval might be sliioghtly smaller or larger

Assymptotic intervals may not be perfect either, although they should get better as N increases

Your task: follow the "How well are we doing?" section

Standard Errors

We used the **standard error** to calculate the assmyptotic confidence. But we could use it to summarise the uncertainty in a single parameter

Standard Deviations of Statistics: s

Binomial variance of n: $Var(n|N, p) = Np(1-p)$

Our statistic: n/N

$Var(n/N) = 1/N^2 Var(n) = p(1-p)/N$

Standard error:

$s = \sqrt{p(1-p)/N}$

Standard Errors: Your Turn

Guess what?

Your task: follow the "Standard Errors " section