

Maximum likelihood:  
a bit of context

# Outline

Step back – the bigger picture

From data to maximum likelihood estimation  
(and back again):

- Population and sample
- Model choice
- Parameter estimation
- Uncertainty
- Interpretation

# Steps of modelling (week 2 recap)

1. Choose a model for your data
2. Get estimates of the parameters
3. Quantify uncertainty in the estimates
4. Interpret the results

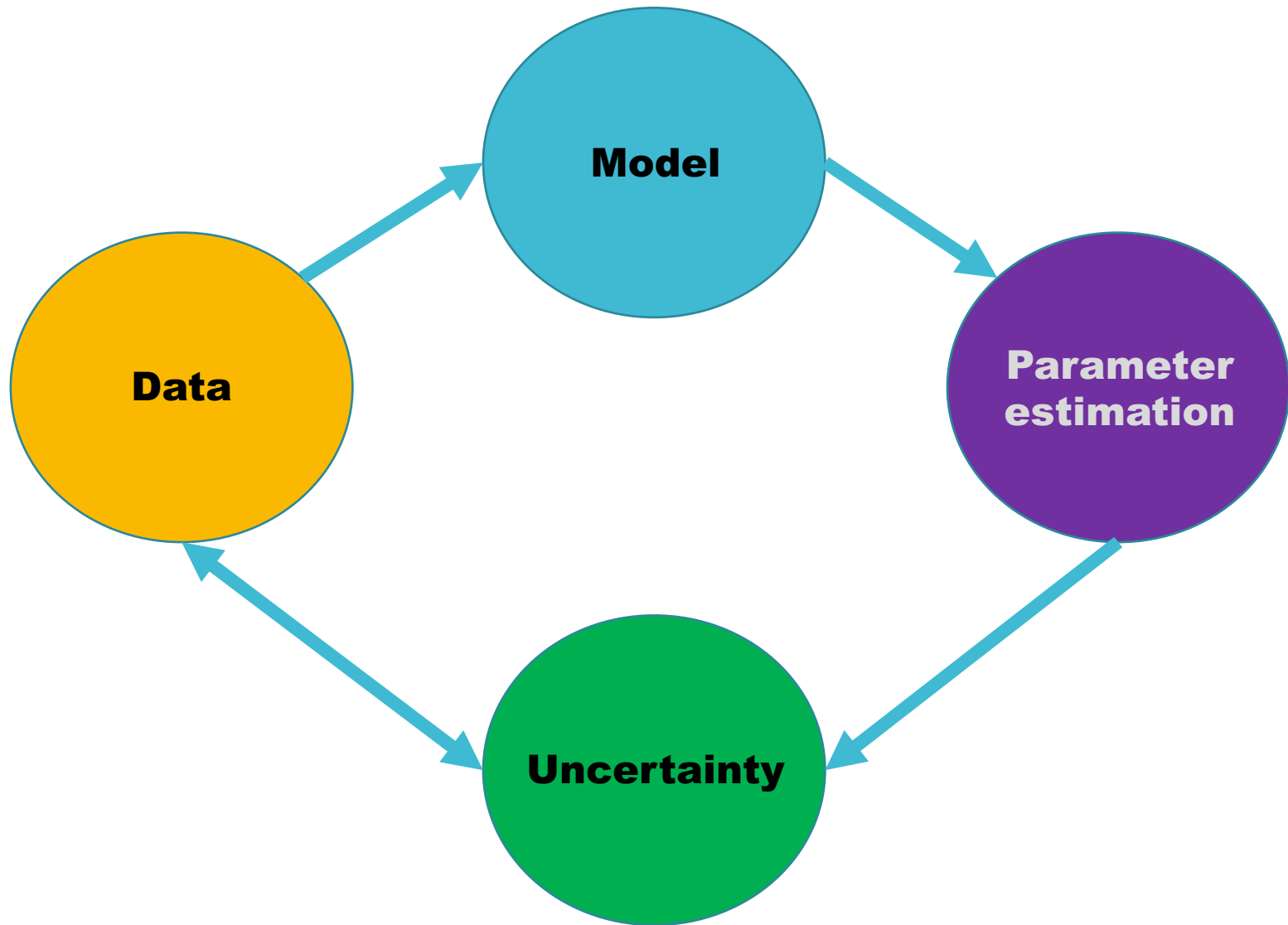
# Steps of modelling (week 2 recap)

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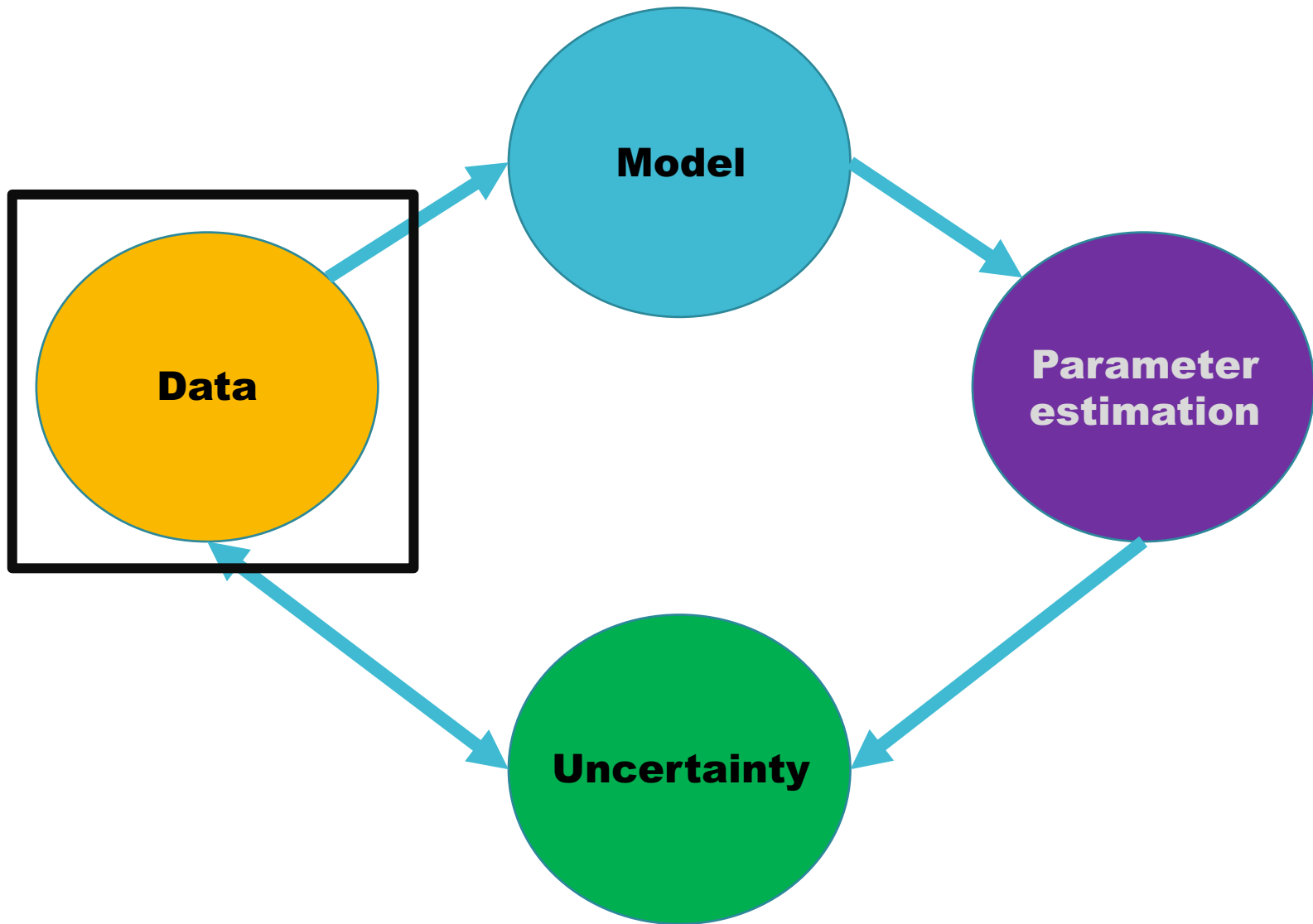
**Maximum likelihood**



# Schematic

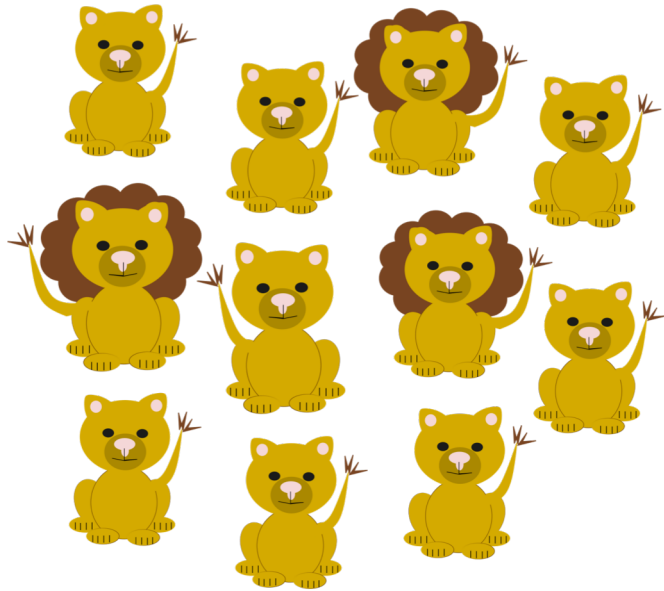


# Schematic



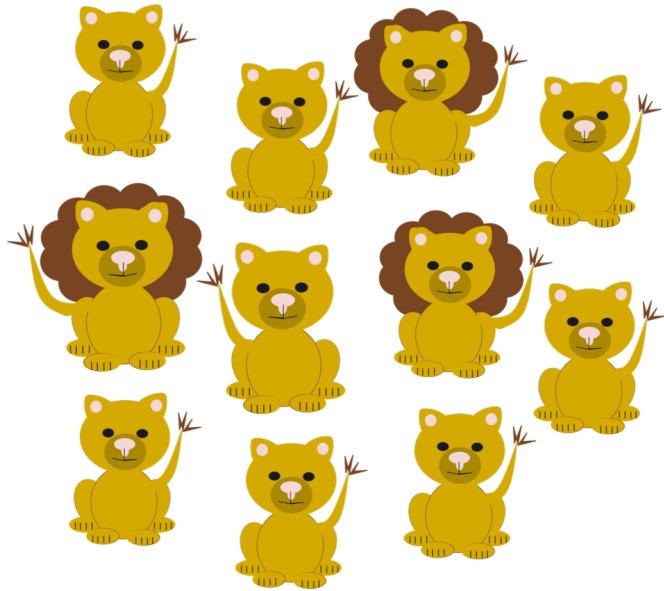
# Data in statistical modelling

## Example of lions



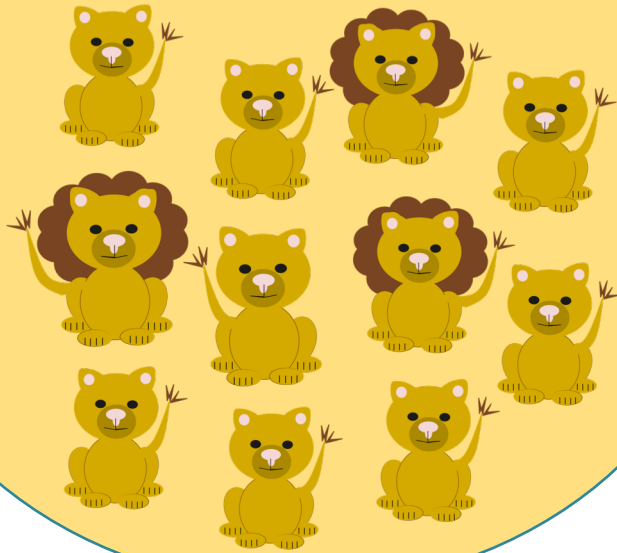
# Data in statistical modelling

## Example of lions

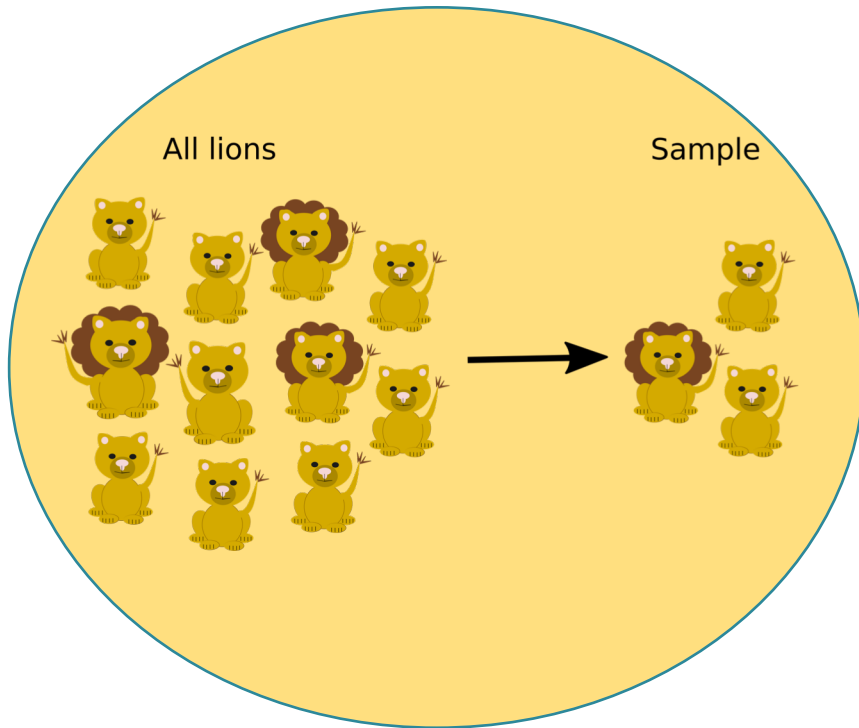


# Data in statistical modelling

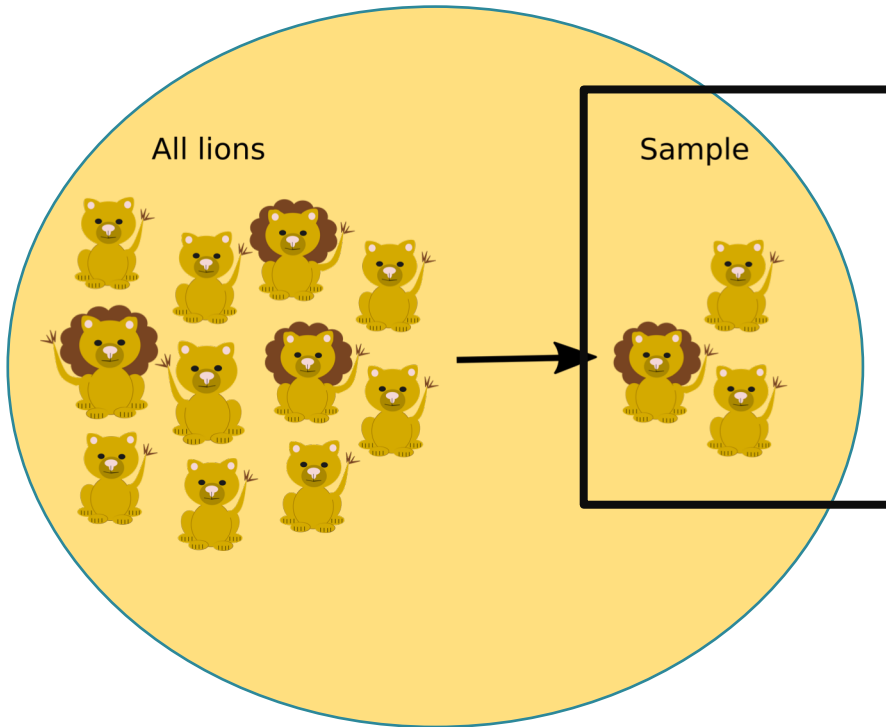
All lions



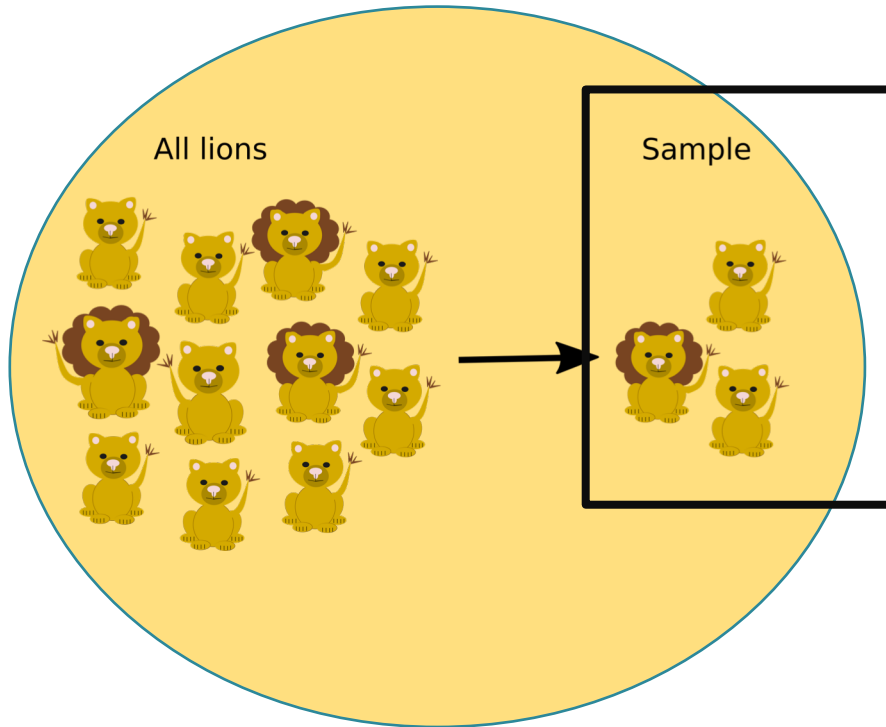
# Data in statistical modelling



# Data in statistical modelling



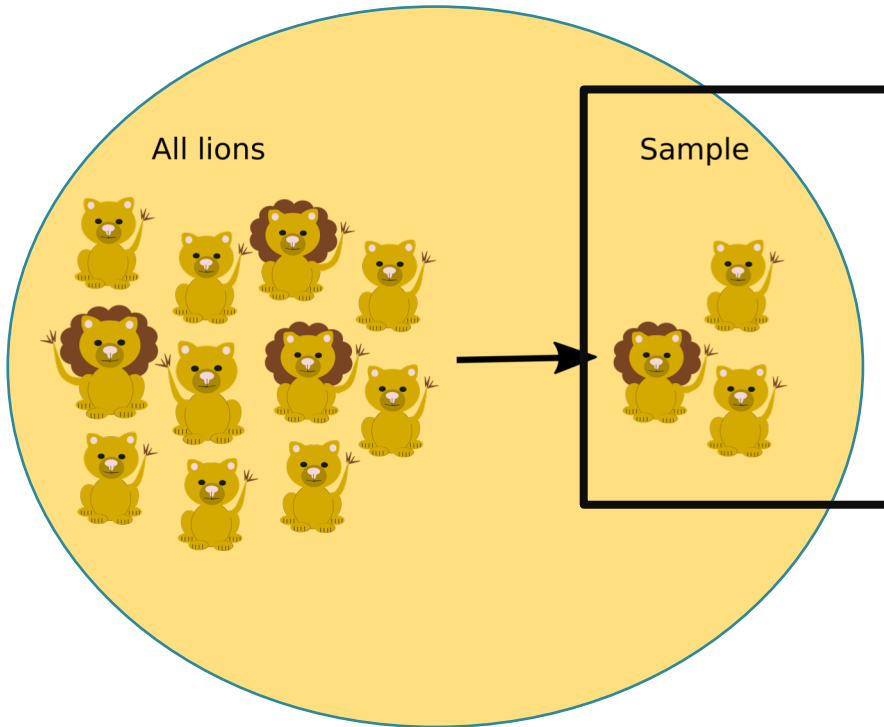
# Data in statistical modelling



Count number of  
lions in prides  
(100 prides here)



# Data in statistical modelling



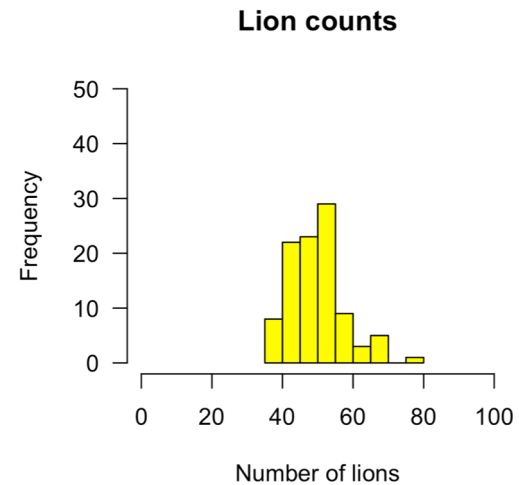
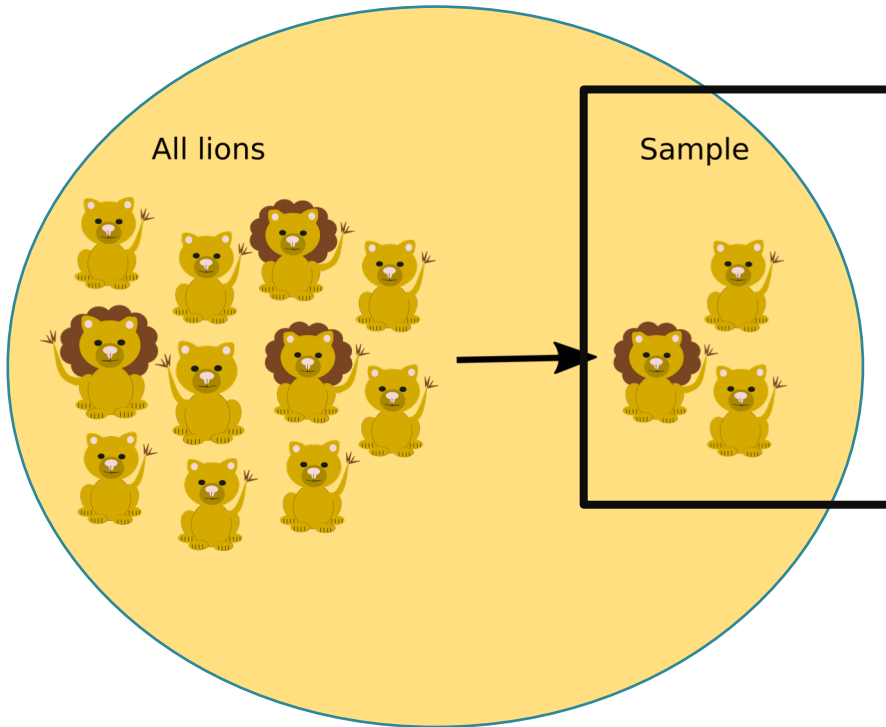
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**This becomes  
our data**

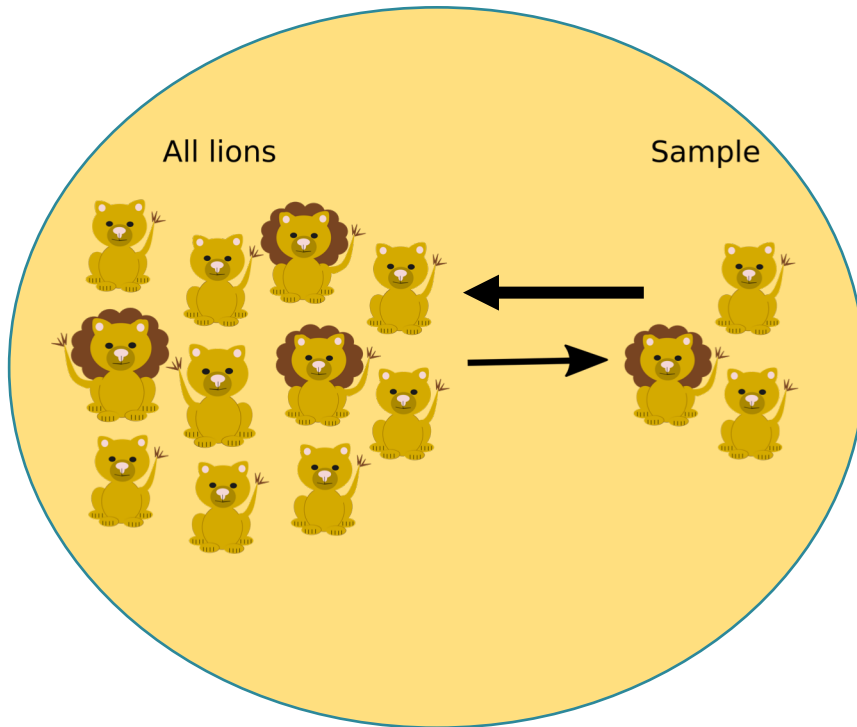
# Data in statistical modelling

Count number of lions in prides (100 prides here)

This becomes our data

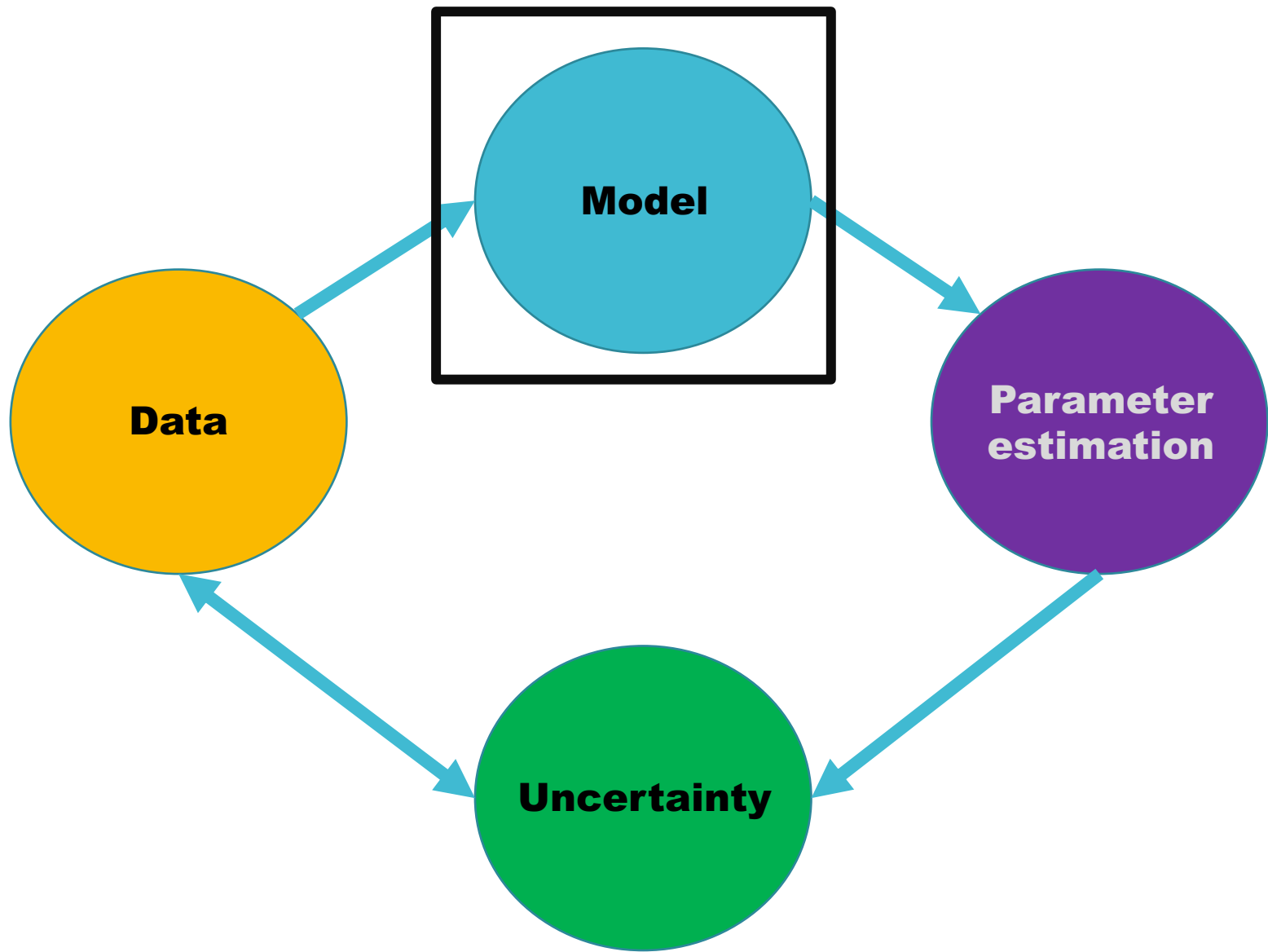


# Data in statistical modelling

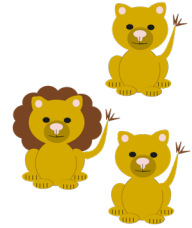


Want to say something  
about total population of  
lions

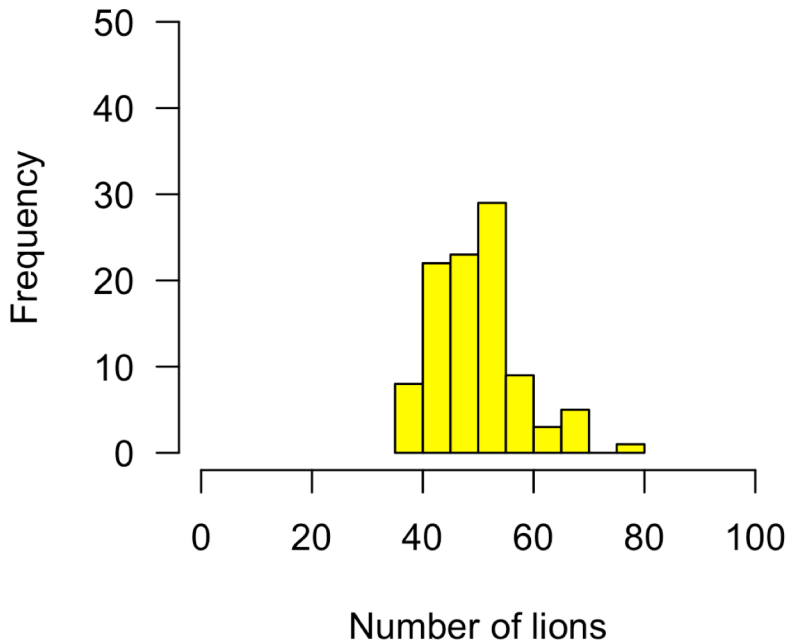
Need a model!



# Choosing a model

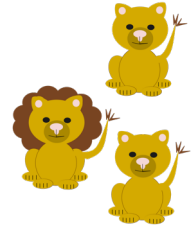


**Lion counts**

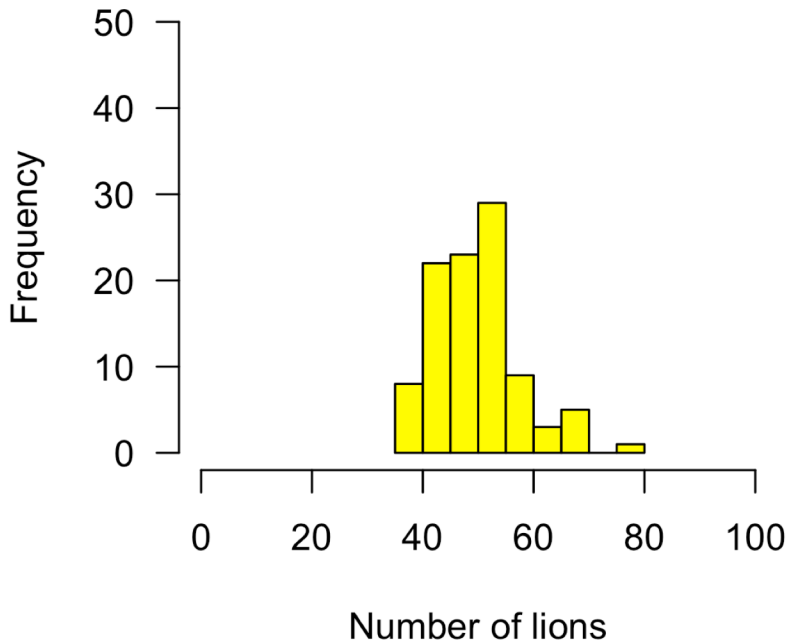


- Counts of numbers of lions
- Cannot be below 0
- Cannot have half lions (whole numbers only)

# Choosing a model



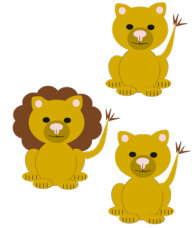
**Lion counts**



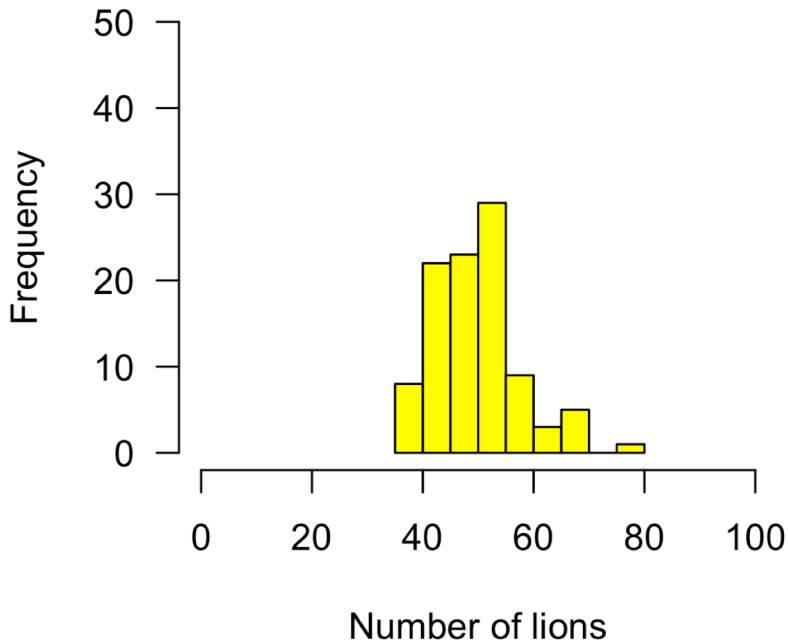
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## **Poisson distribution**

# Choosing a model

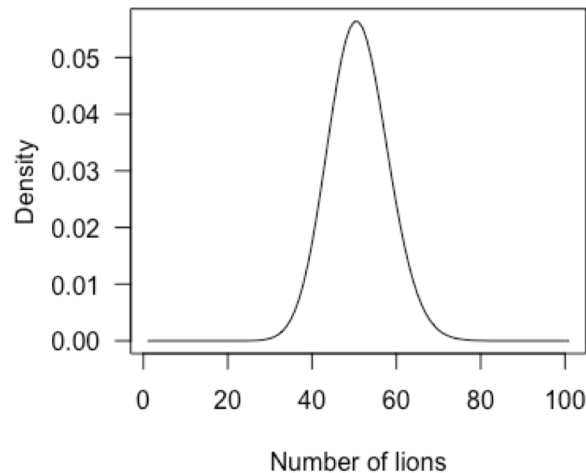


## Lion counts



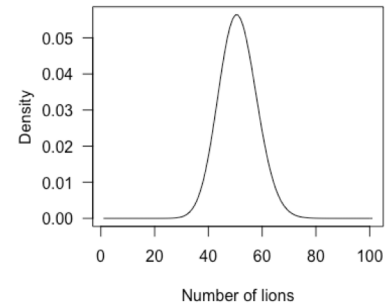
- Counts of numbers of lions
- Cannot be below 0
- Cannot have half lions (whole numbers only)

## Poisson distribution



# Any model

Our example = **Poisson distribution**



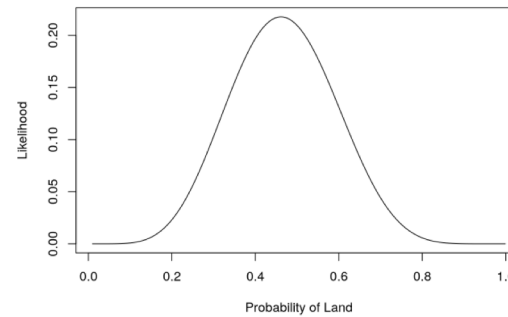


# Any model

Our example = Poisson distribution

**But works for any model**

E.g. Binomial distribution (land and sea)



# Any model

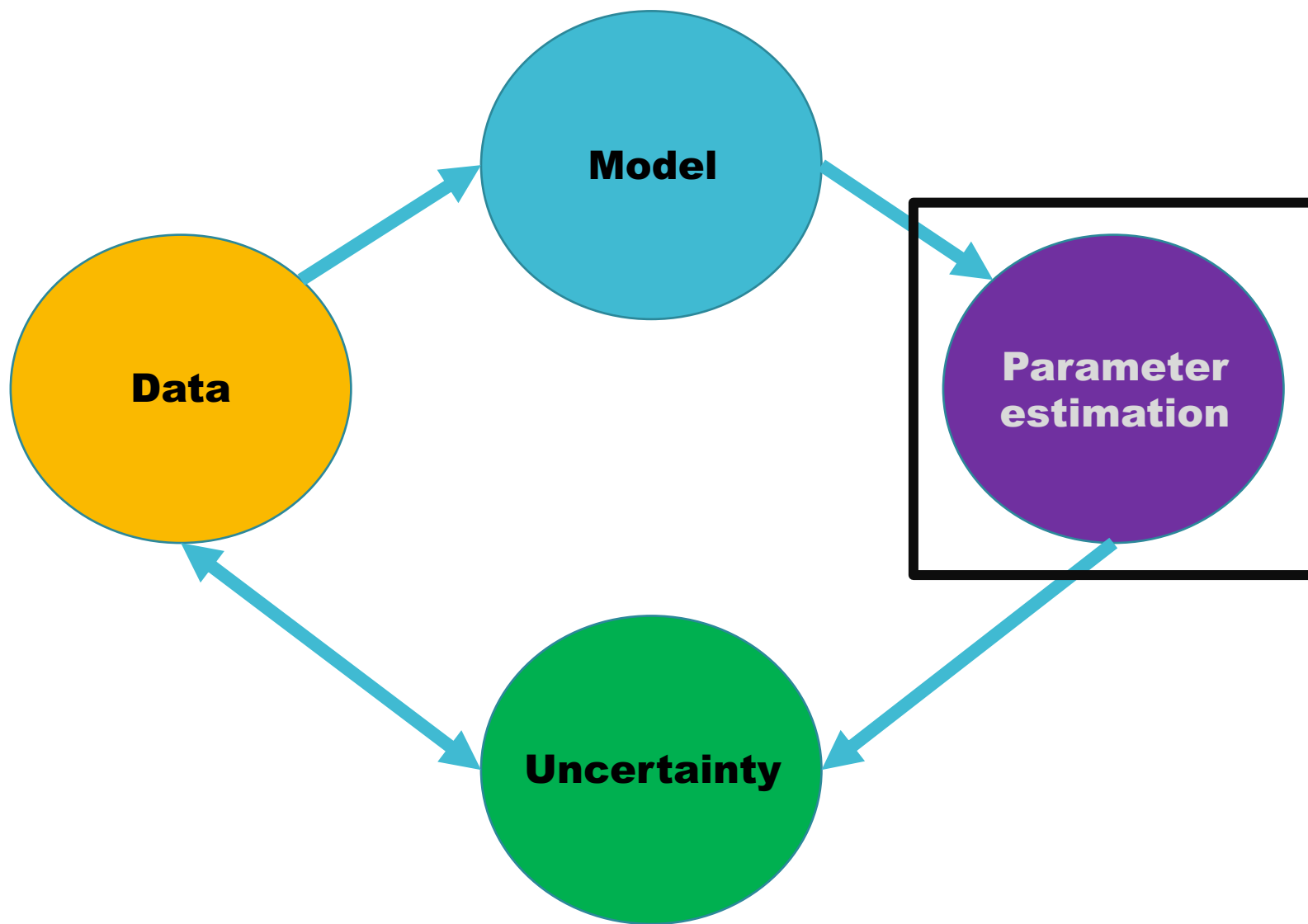
Our example = Poisson distribution

**But works for any model**

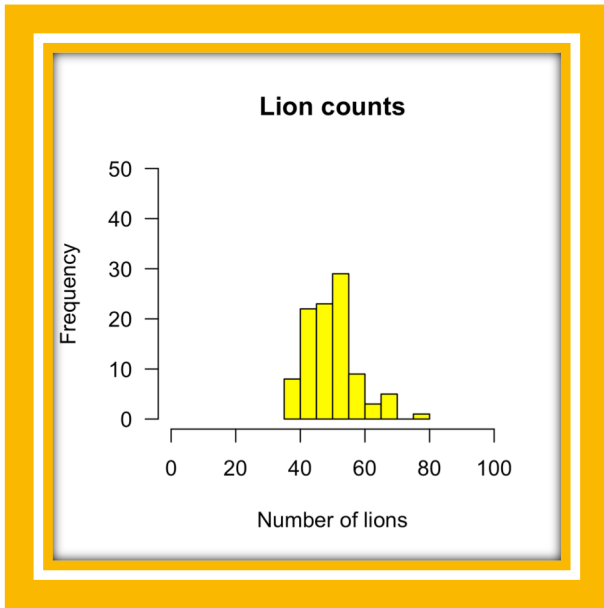
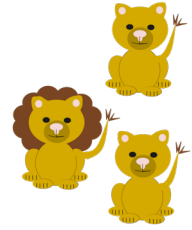
E.g. Binomial distribution (land and sea)

Linear equation (regression – coming soon)

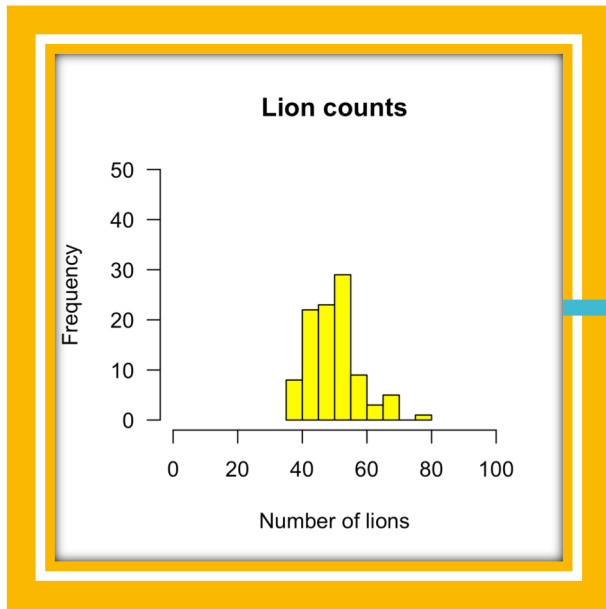
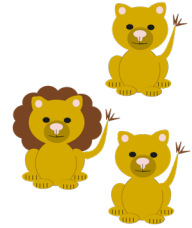
Almost anything



# Parameter estimation

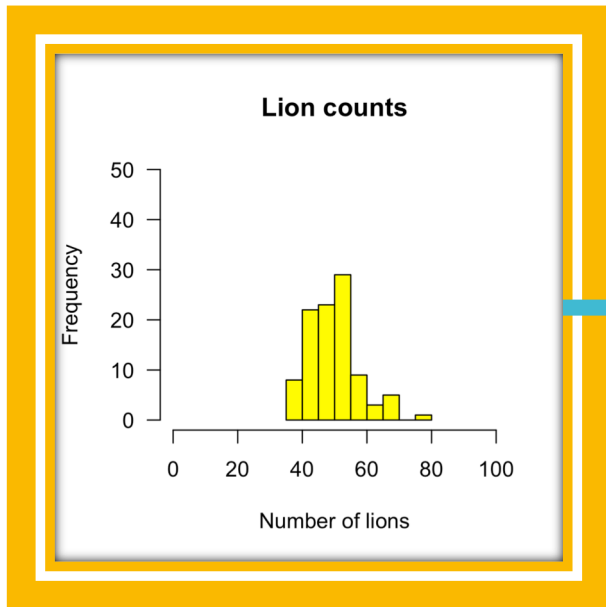
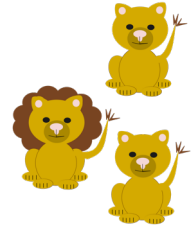


# Parameter estimation



**Poisson  
distribution**

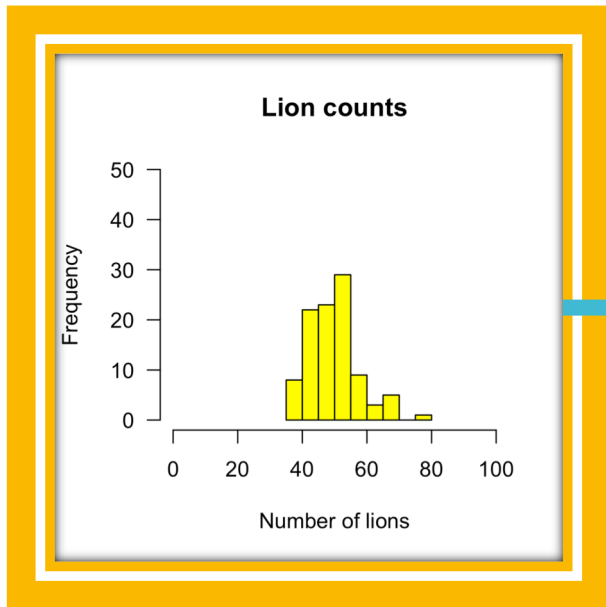
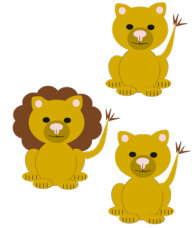
# Parameter estimation



Parameter =  $\lambda$

**Poisson  
distribution**

# Parameter estimation

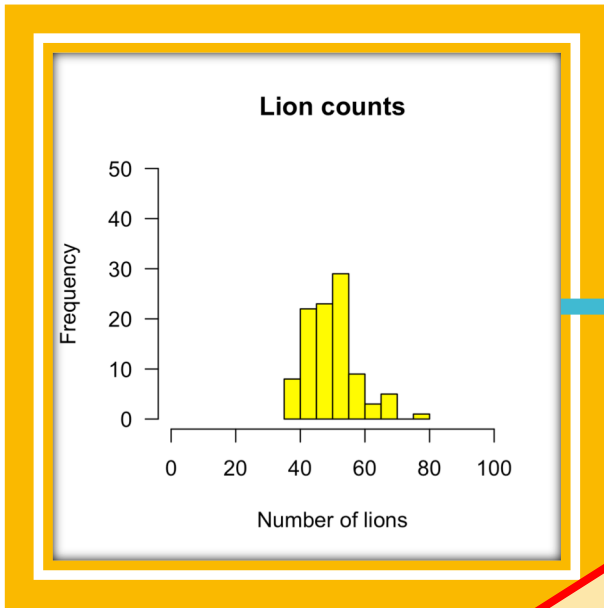
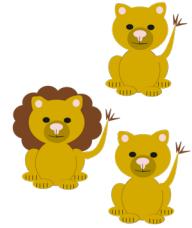


Parameter =  $\lambda$

**Poisson  
distribution**

Want to find the value of the parameter that is most likely to give rise to our observed data

# Parameter estimation



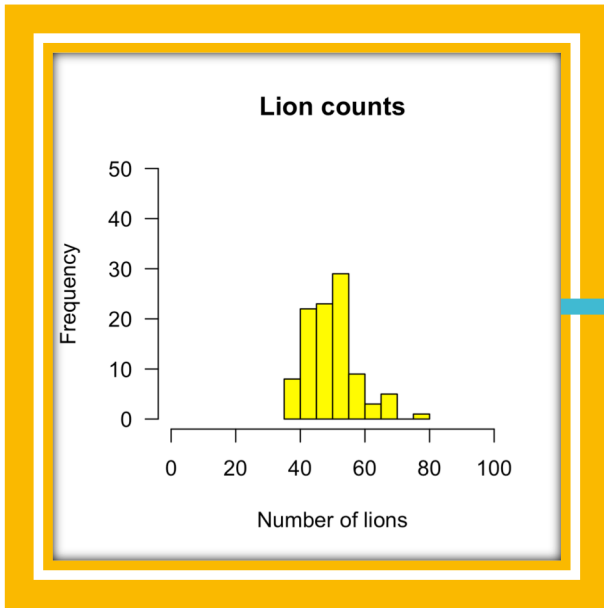
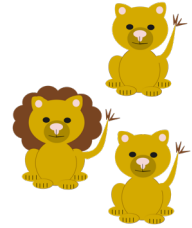
Parameter =  $\lambda$

Poisson distribution

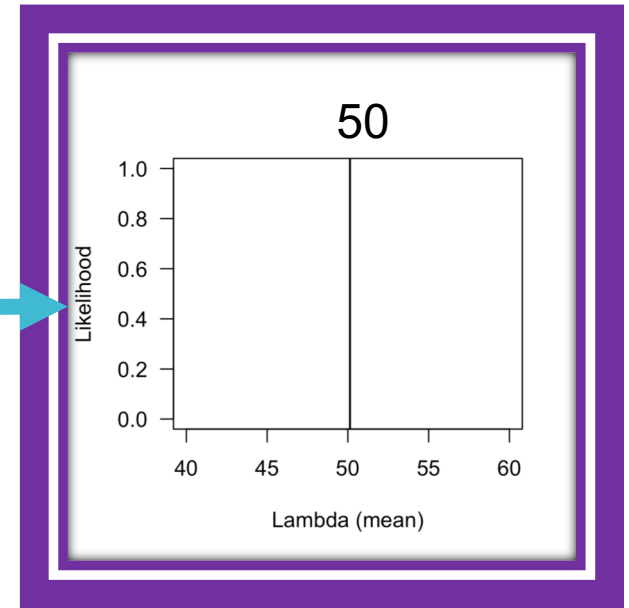
**Maximum likelihood**



# Parameter estimation

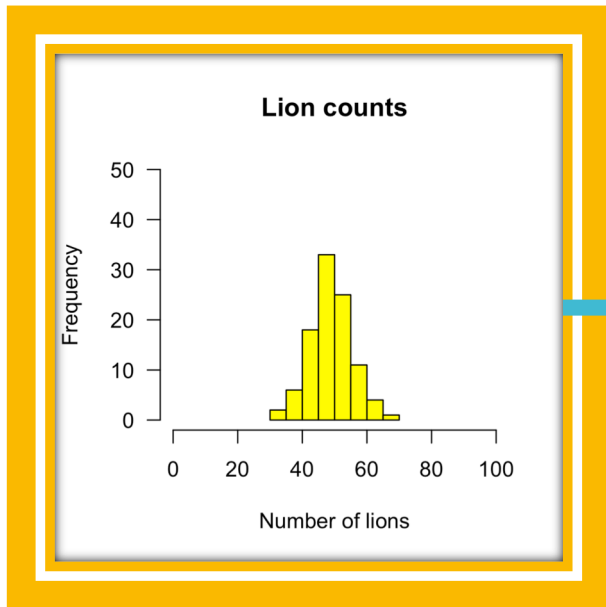
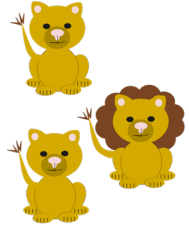


**Poisson  
distribution**



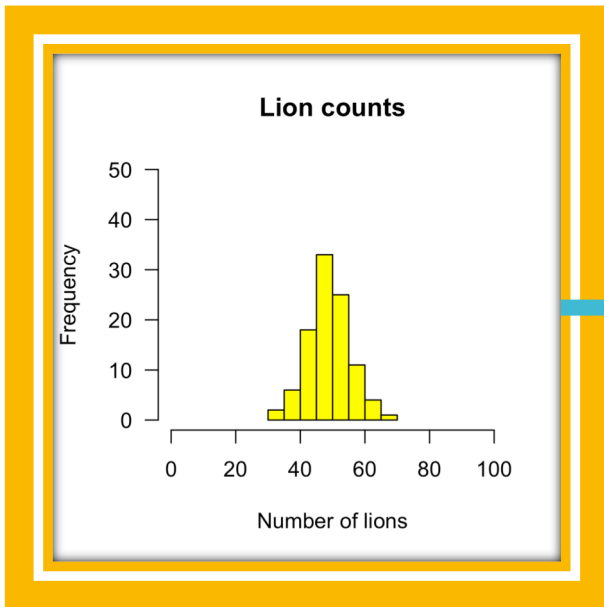
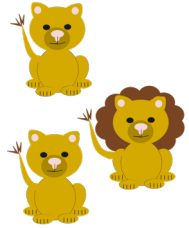
MLE of  
Parameter =  $\hat{\lambda}$

# Parameter estimation

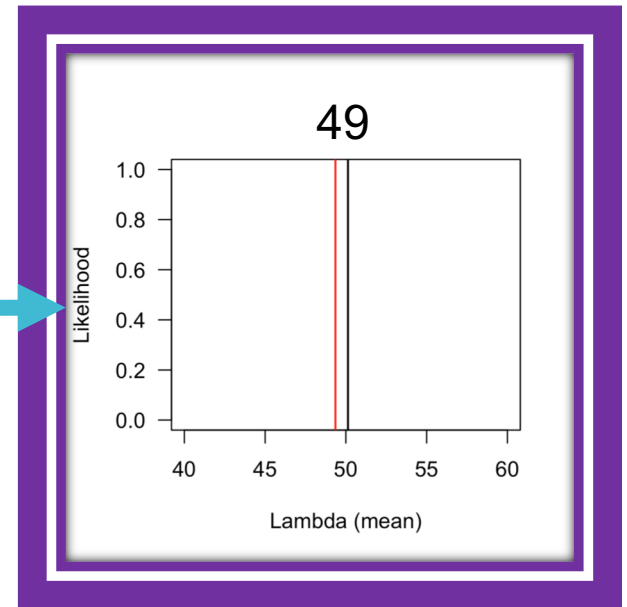


**Poisson  
distribution**

# Parameter estimation

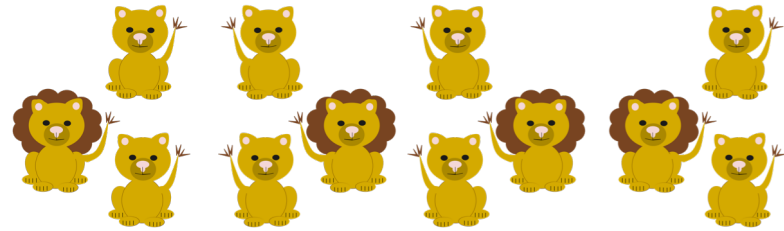


**Poisson  
distribution**

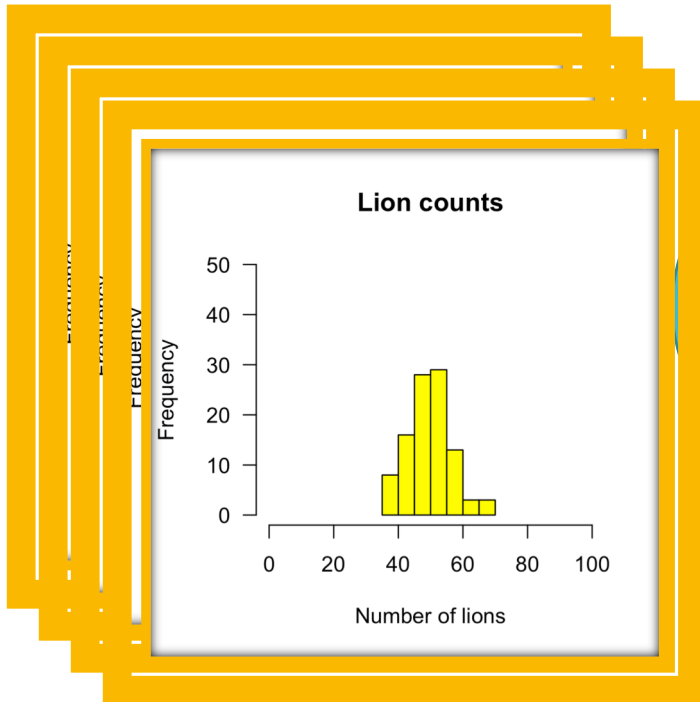


MLE of  
Parameter =  $\hat{\lambda}$

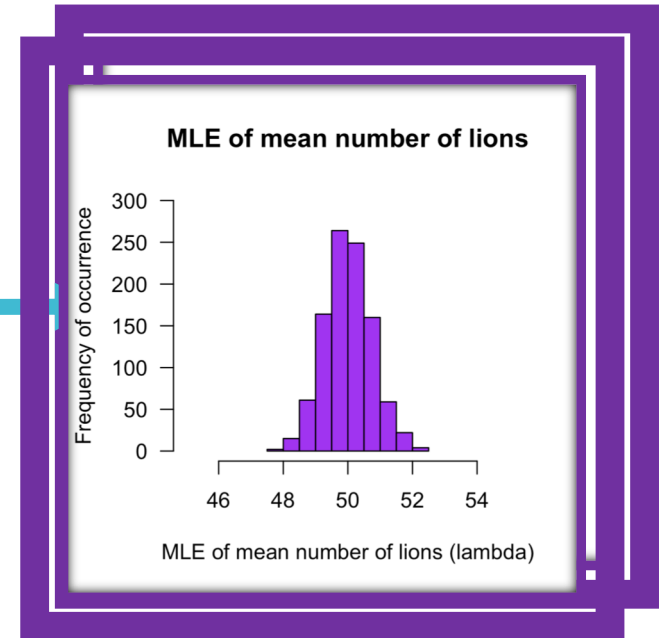
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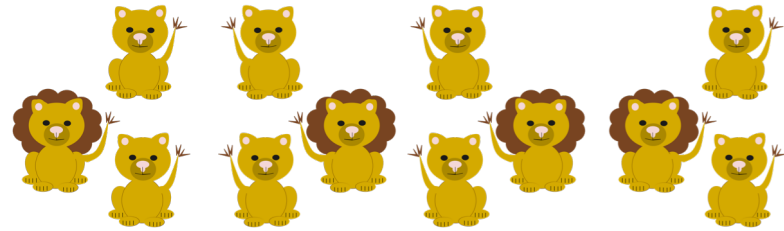
MLE of  
Parameter =  $\hat{\lambda}$



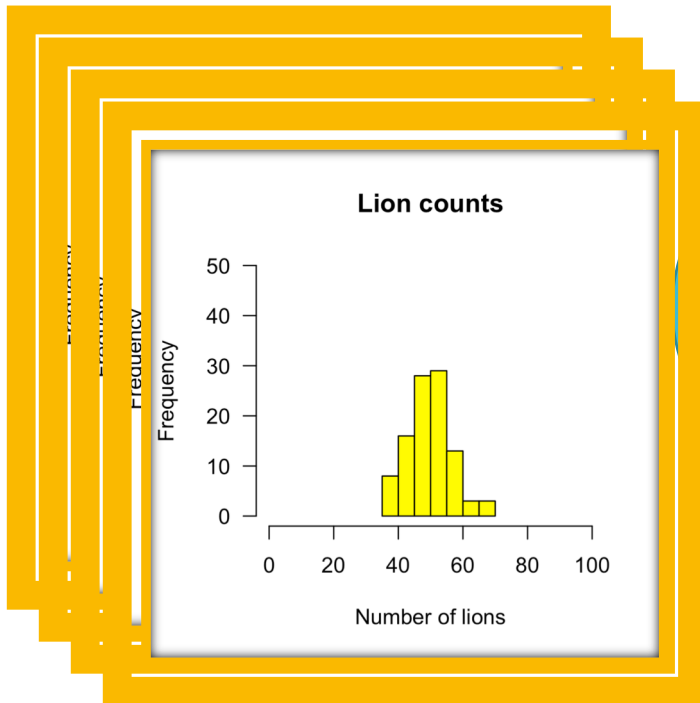
**Poisson  
distribution**



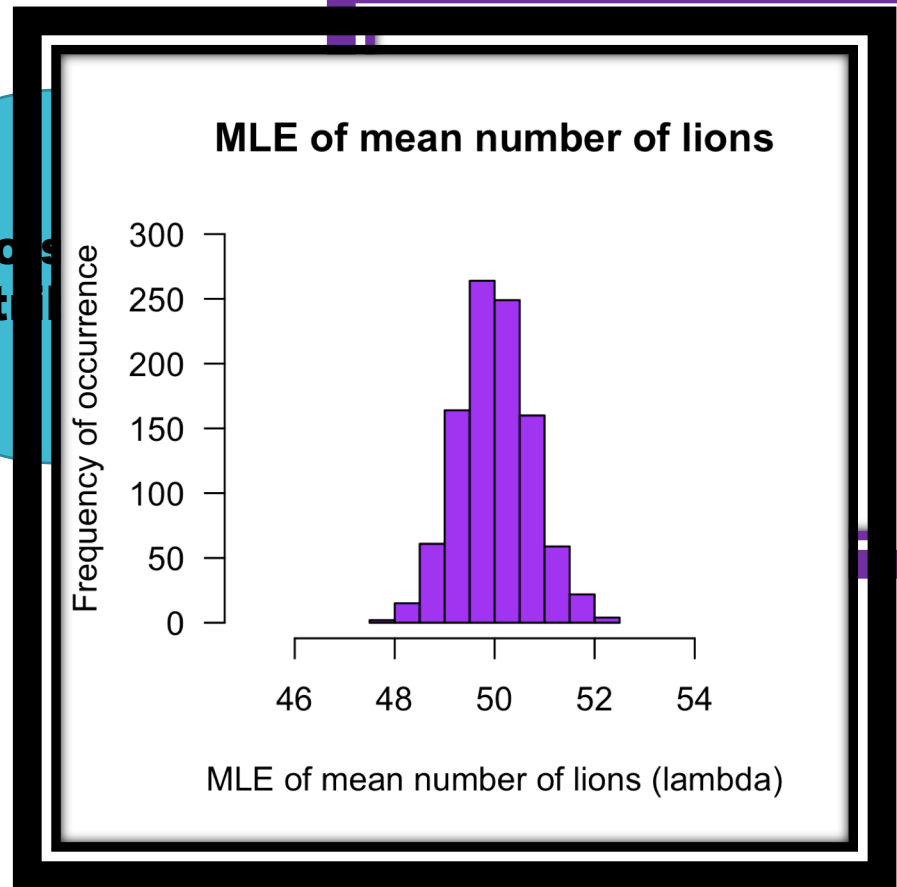
# Parameter estimation



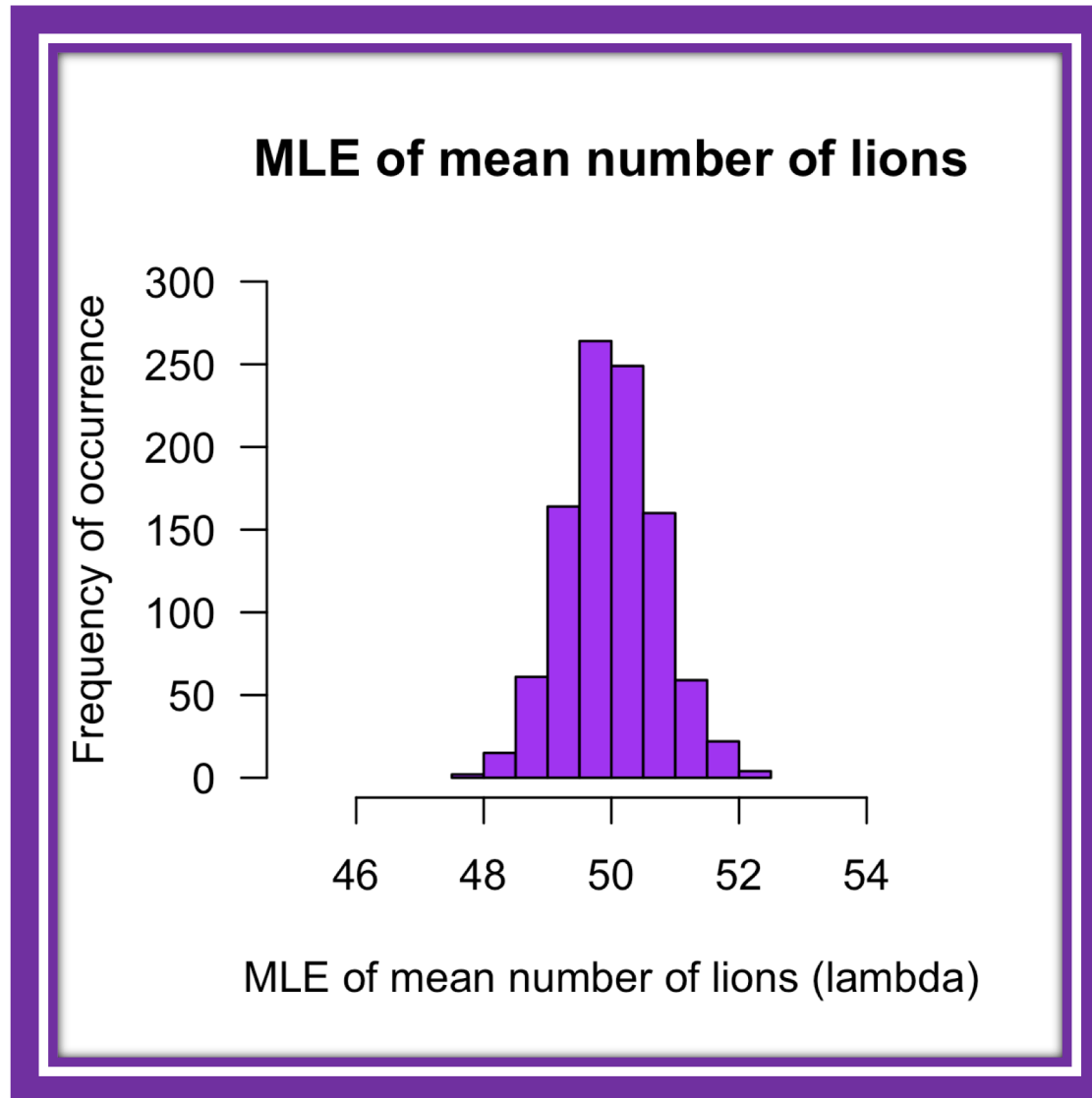
MLE of  
Parameter =  $\hat{\lambda}$



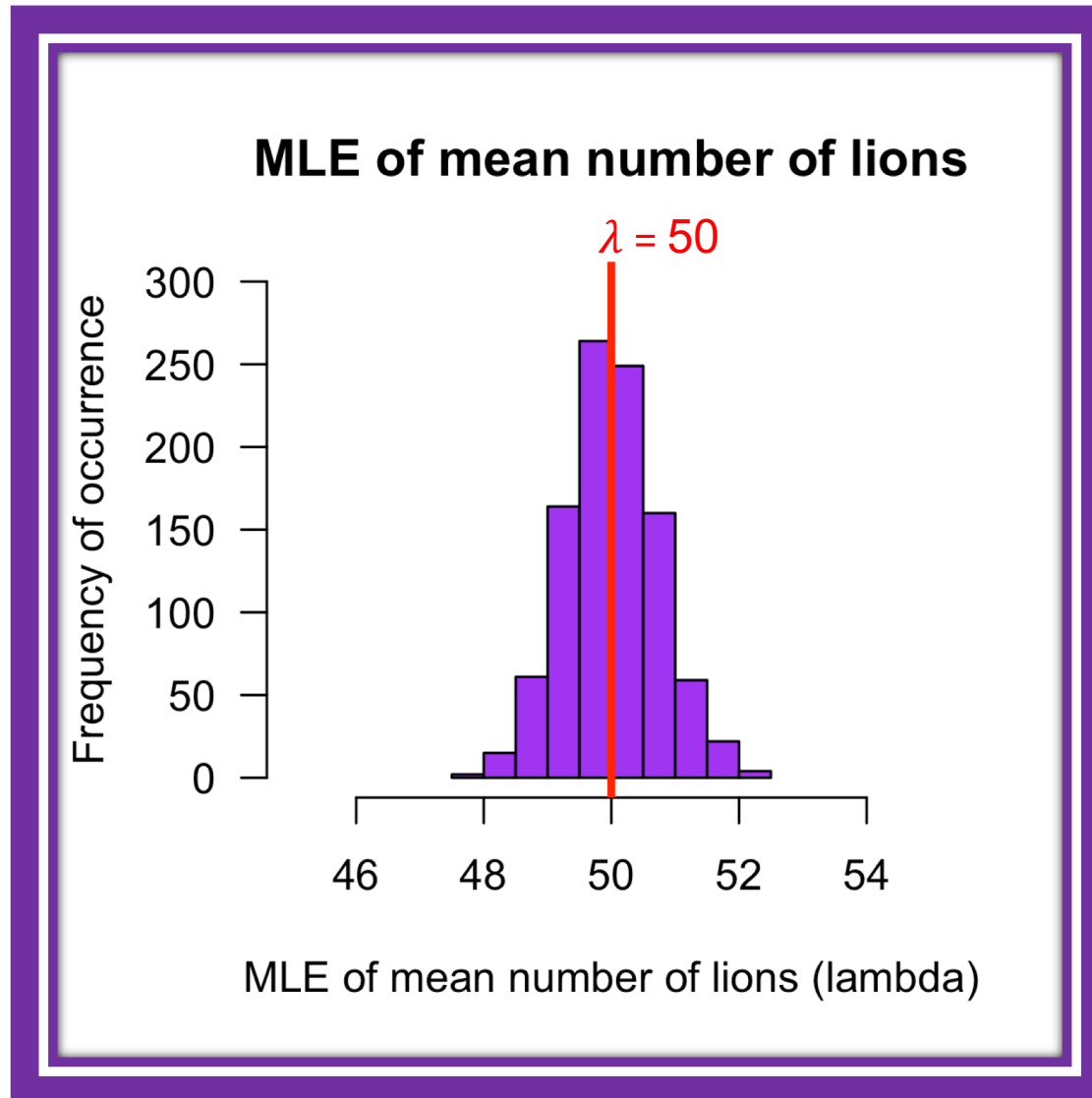
Pos  
dist



# Distribution of $\hat{\lambda}$

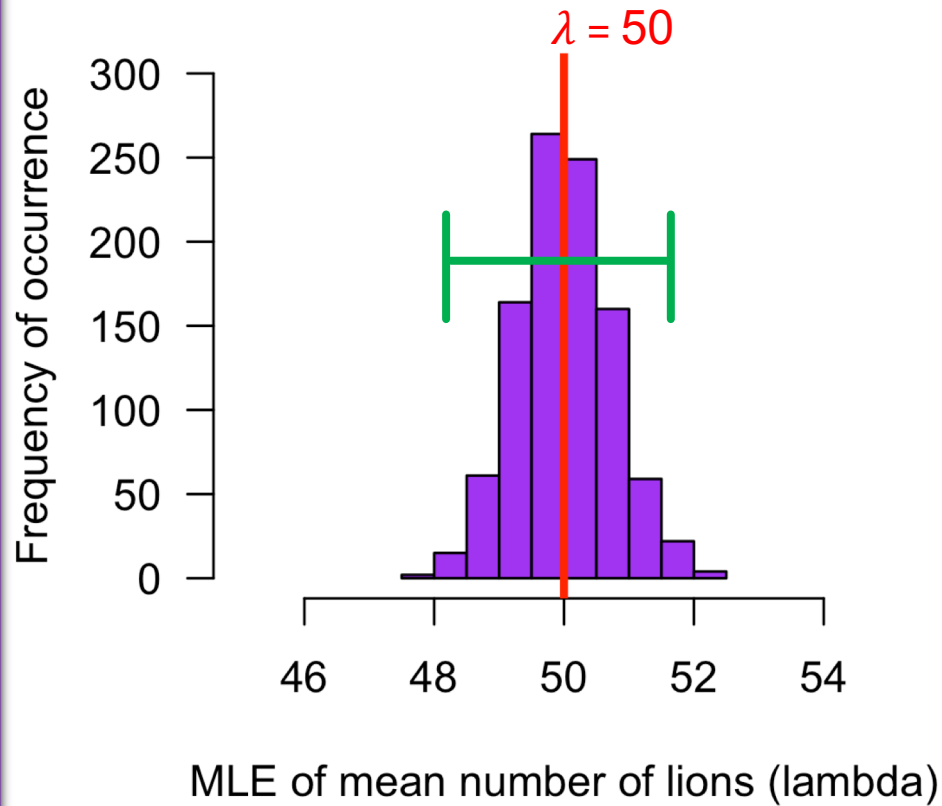


# Distribution of $\hat{\lambda}$



# Distribution of $\hat{\lambda}$

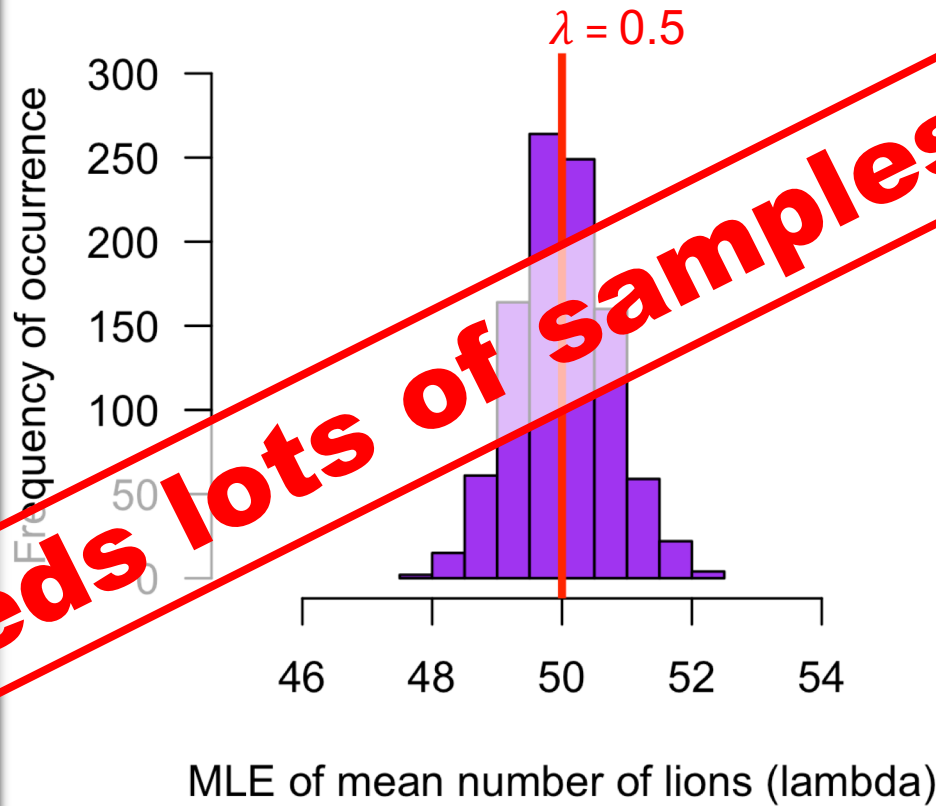
## MLE of mean number of lions





# Distribution of $\hat{\lambda}$

MLE of mean number of lions



**Needs lots of samples**

# Parameter estimation from a single sample

# Parameter estimation from a single sample

We need to:

- represent the distribution of the parameter mathematically
- and based on our single sample of data

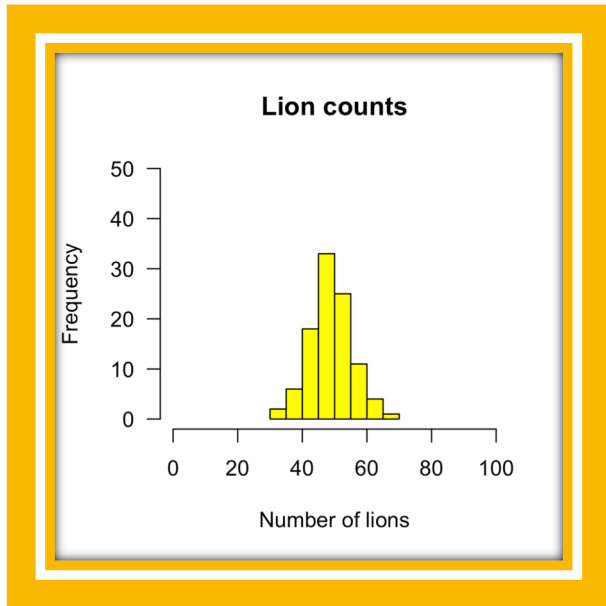
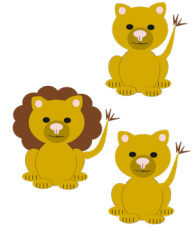
# Parameter estimation from a single sample

We need to:

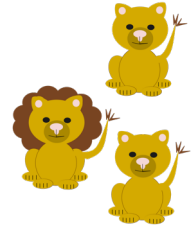
- represent the distribution of the parameter mathematically
- and based on our single sample of data

**This is what we use the likelihood to do**

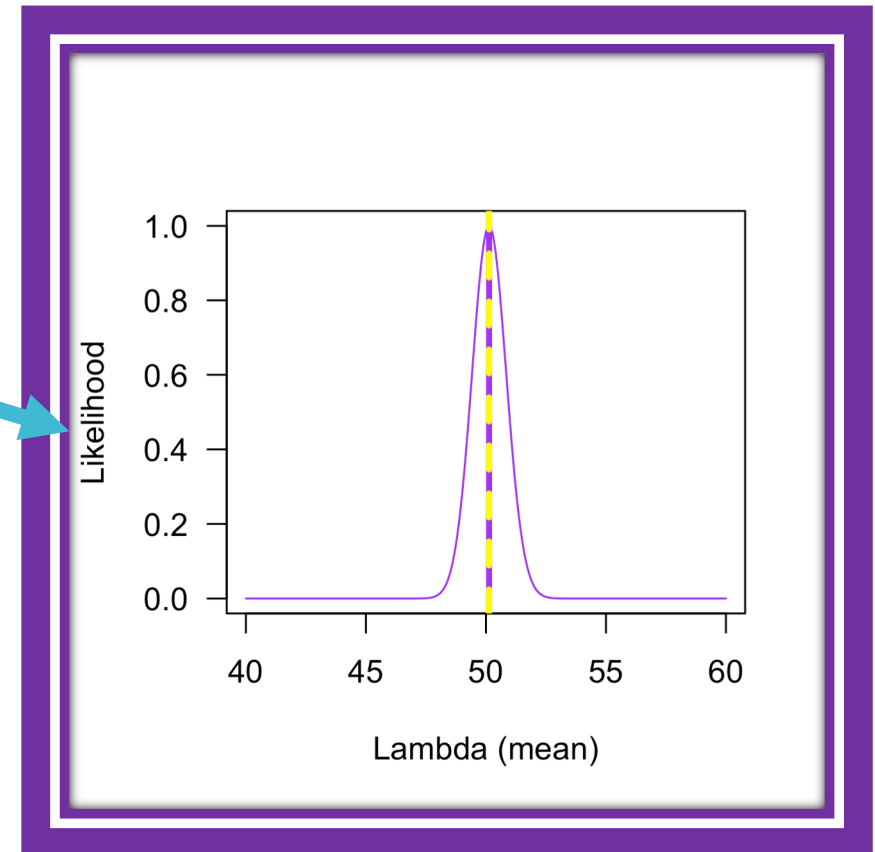
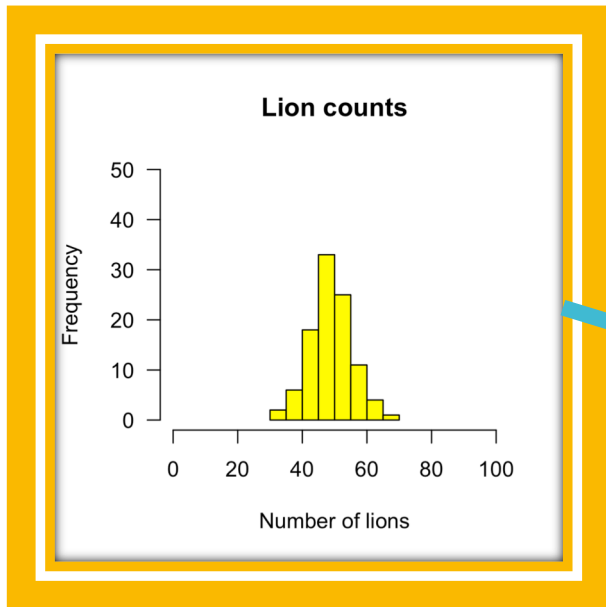
# Parameter estimation from a single sample



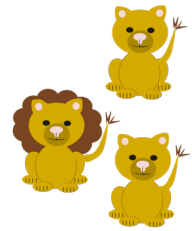
# Parameter estimation from a single sample



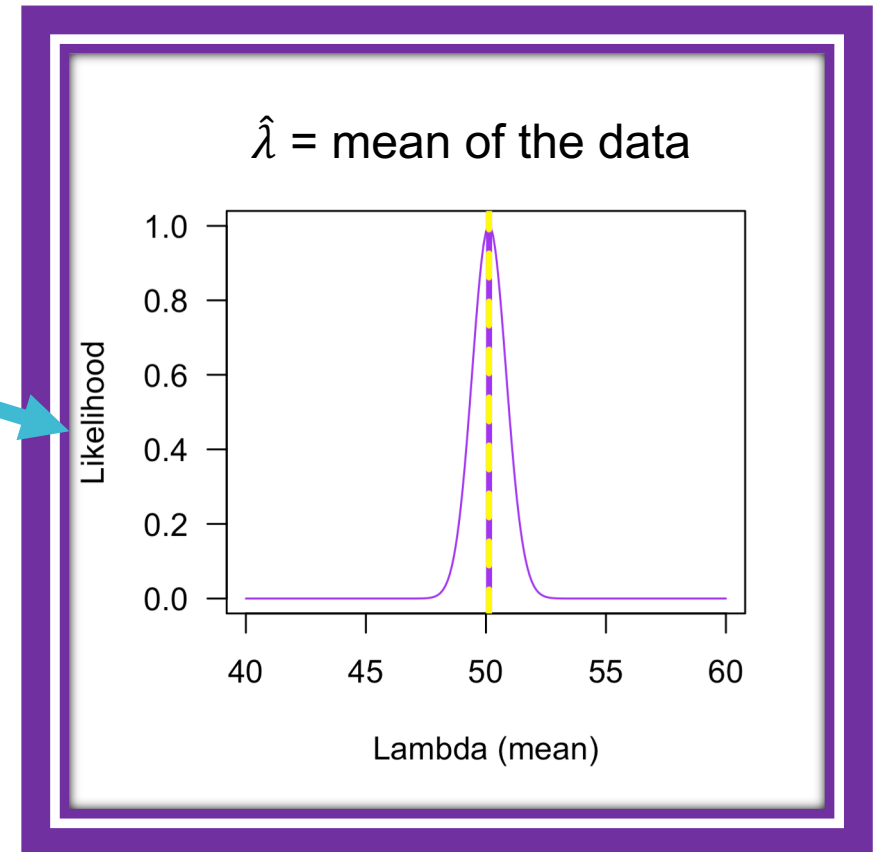
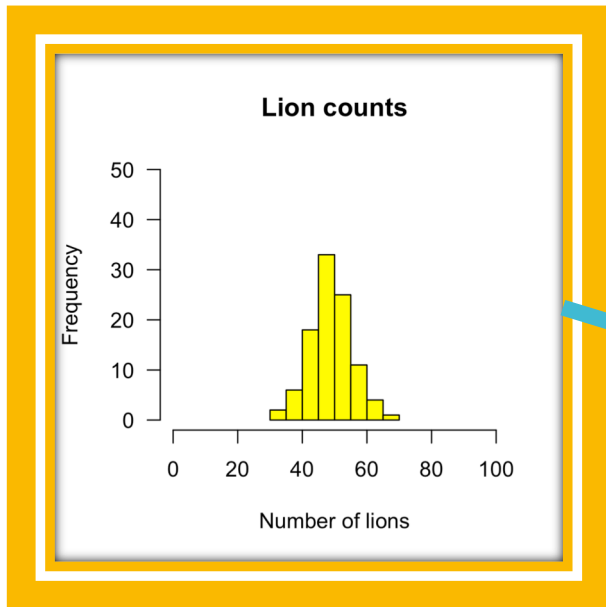
Plot of likelihood  
for different  
values of  $\lambda$



# Parameter estimation from a single sample

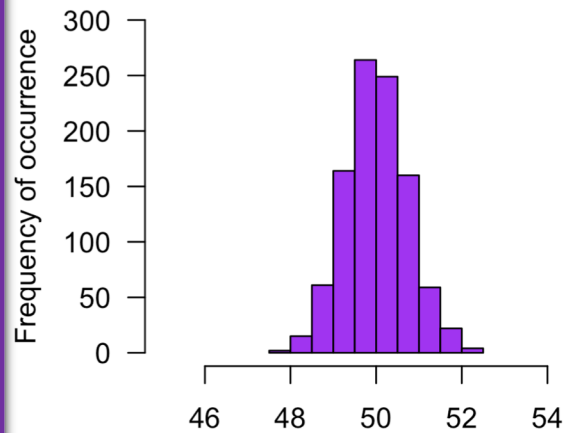


Plot of likelihood  
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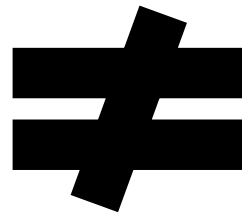


# Parameter estimation from a single sample

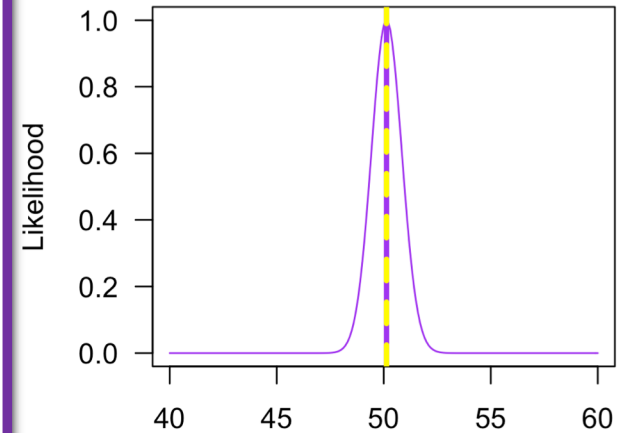
MLE of mean number of lions



MLE of mean number of lions (lambda)

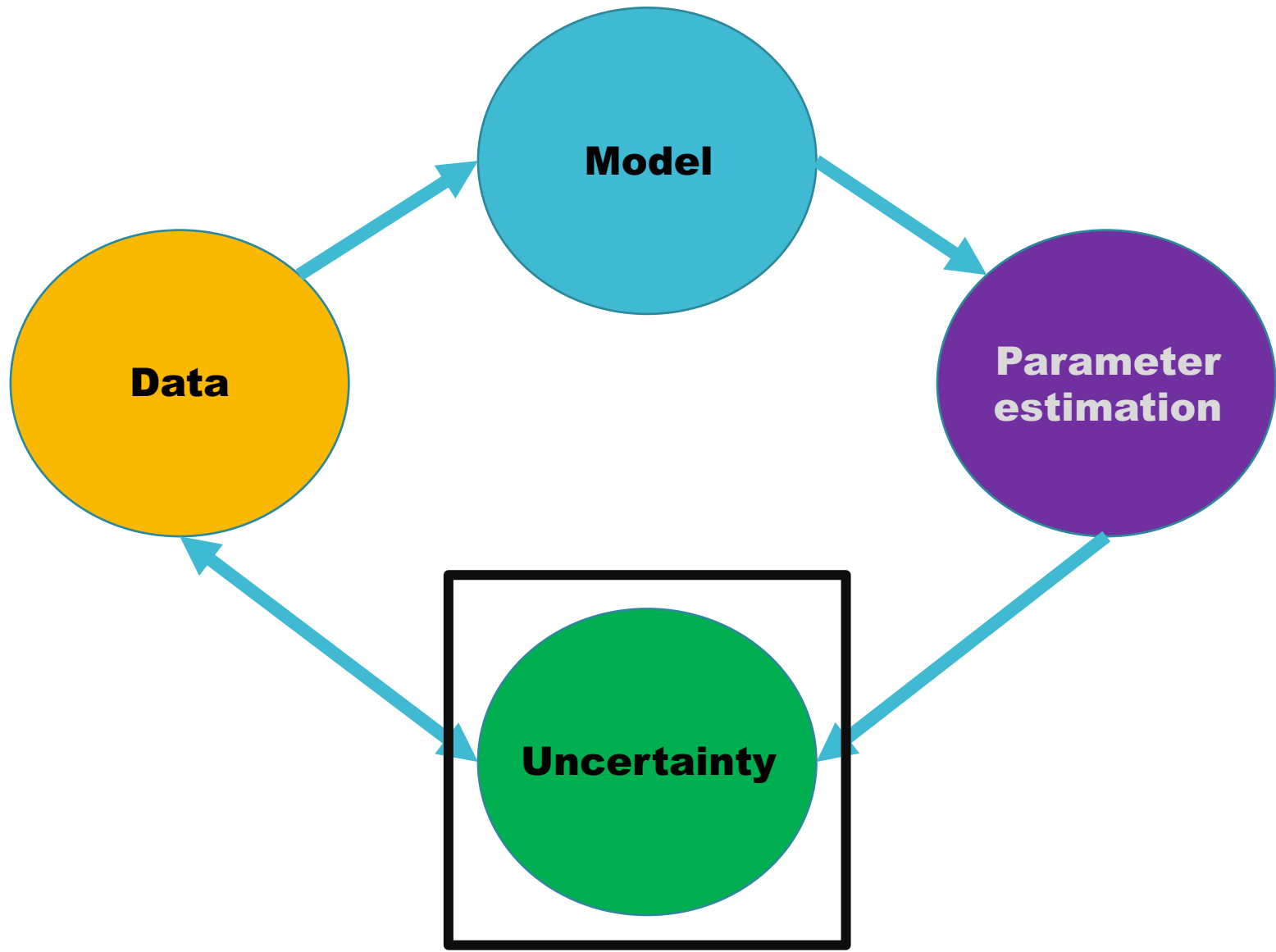


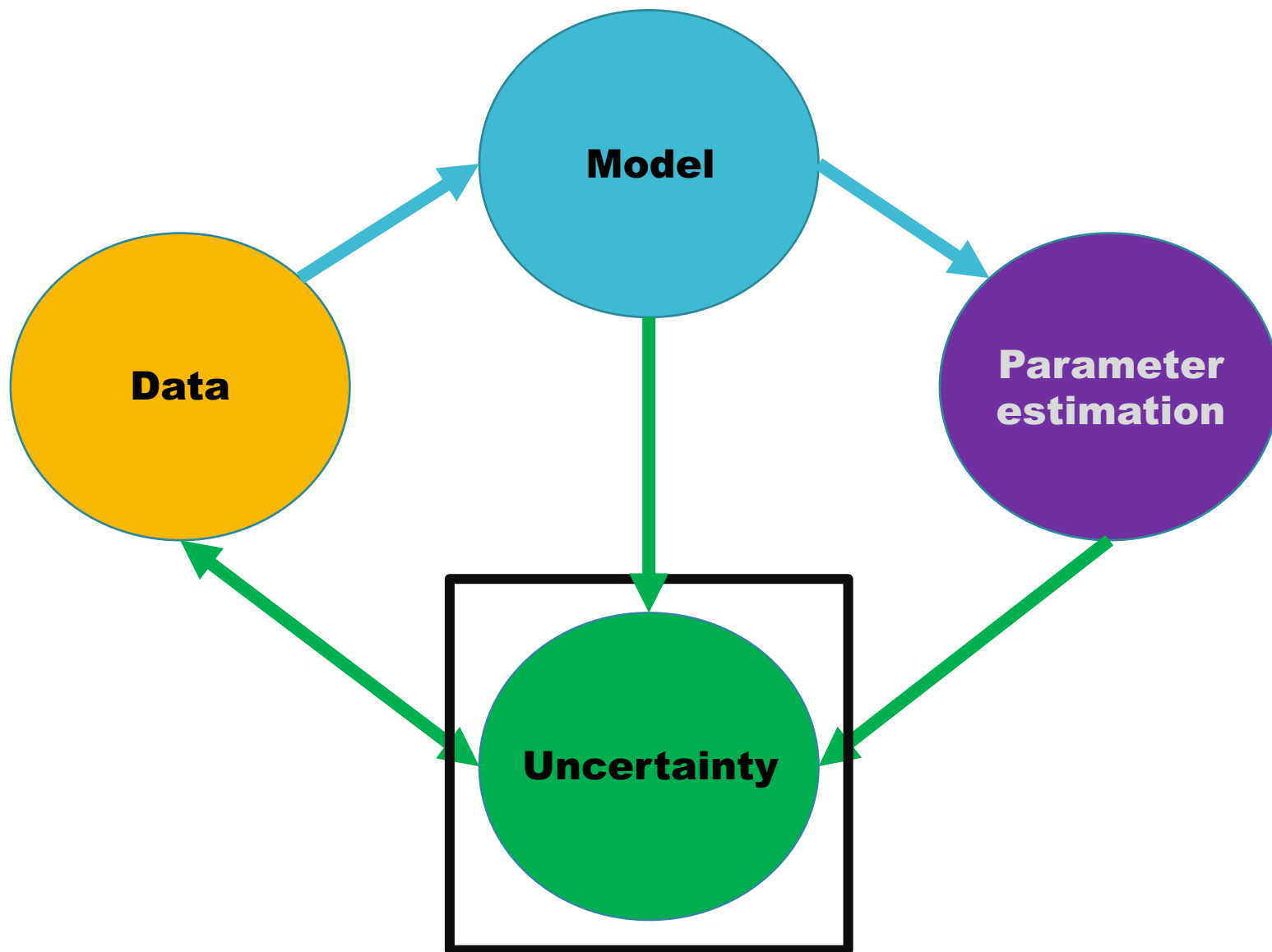
$\hat{\lambda} = \text{mean of the data}$



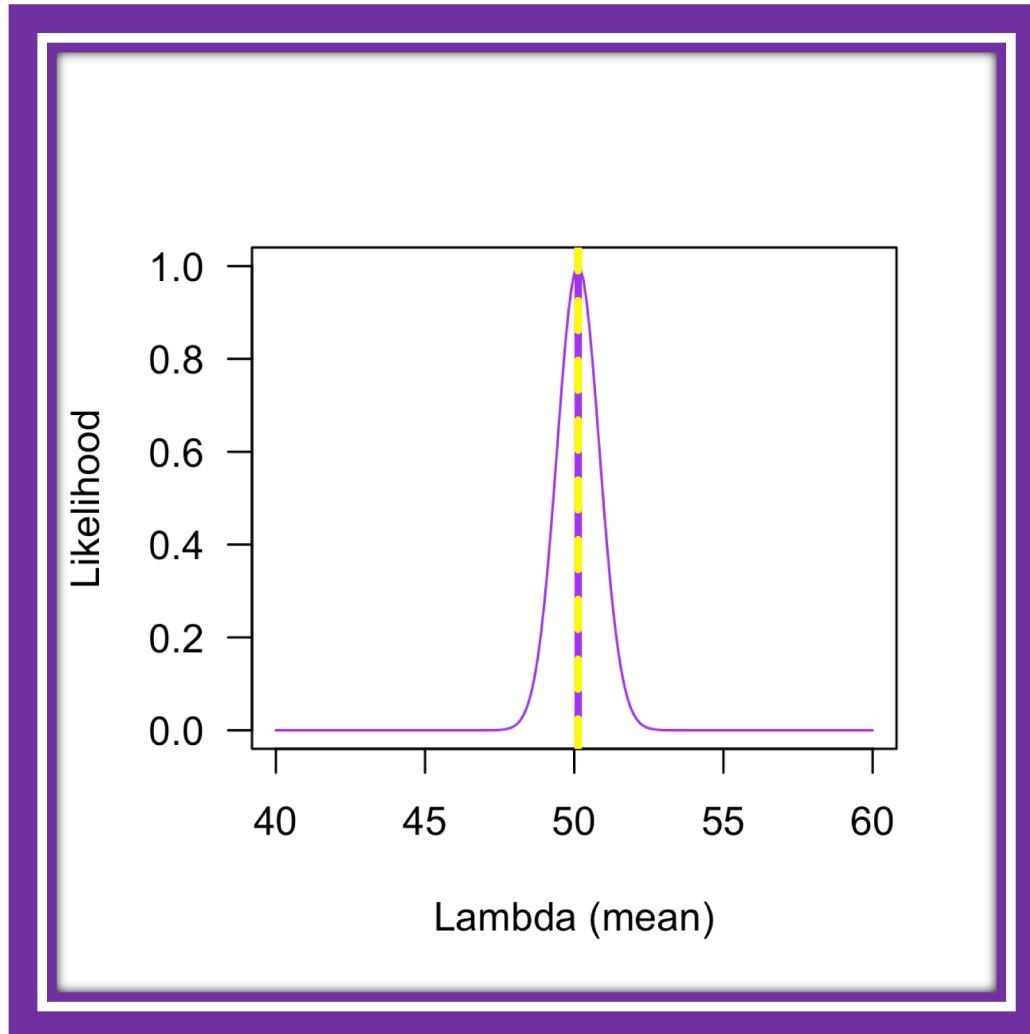
Lambda (mean)



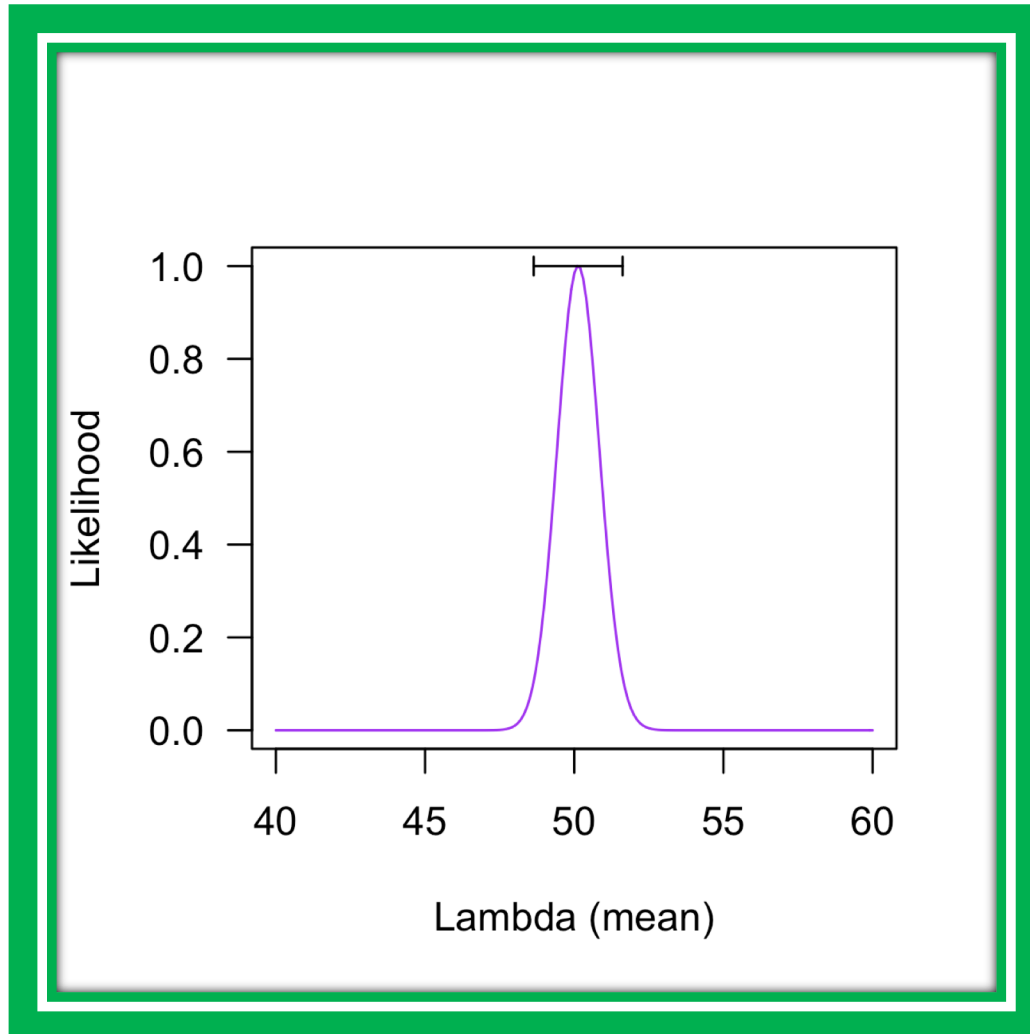




# Uncertainty

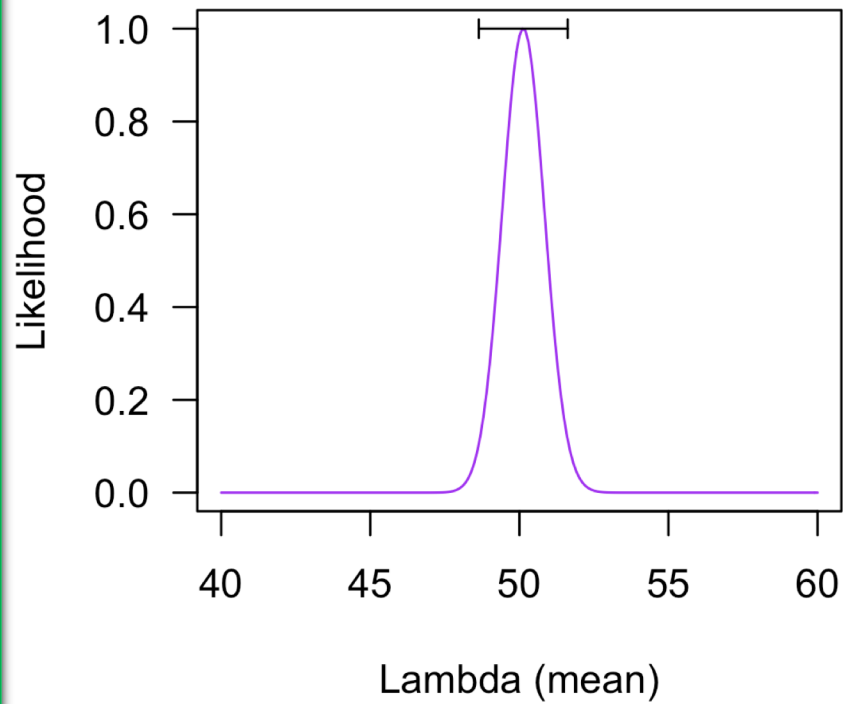


# Uncertainty



# Uncertainty

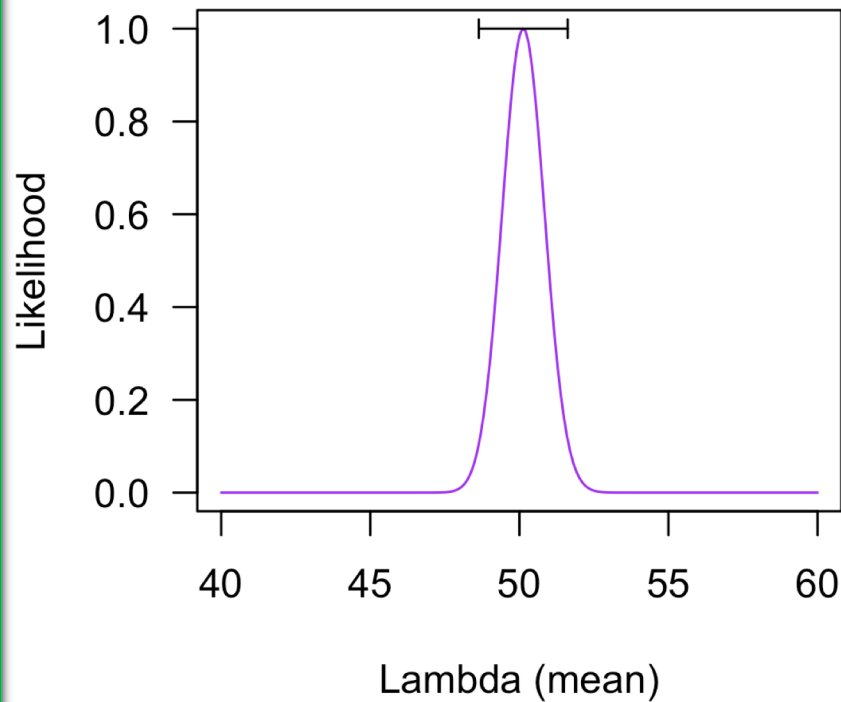
How can we quantify it?



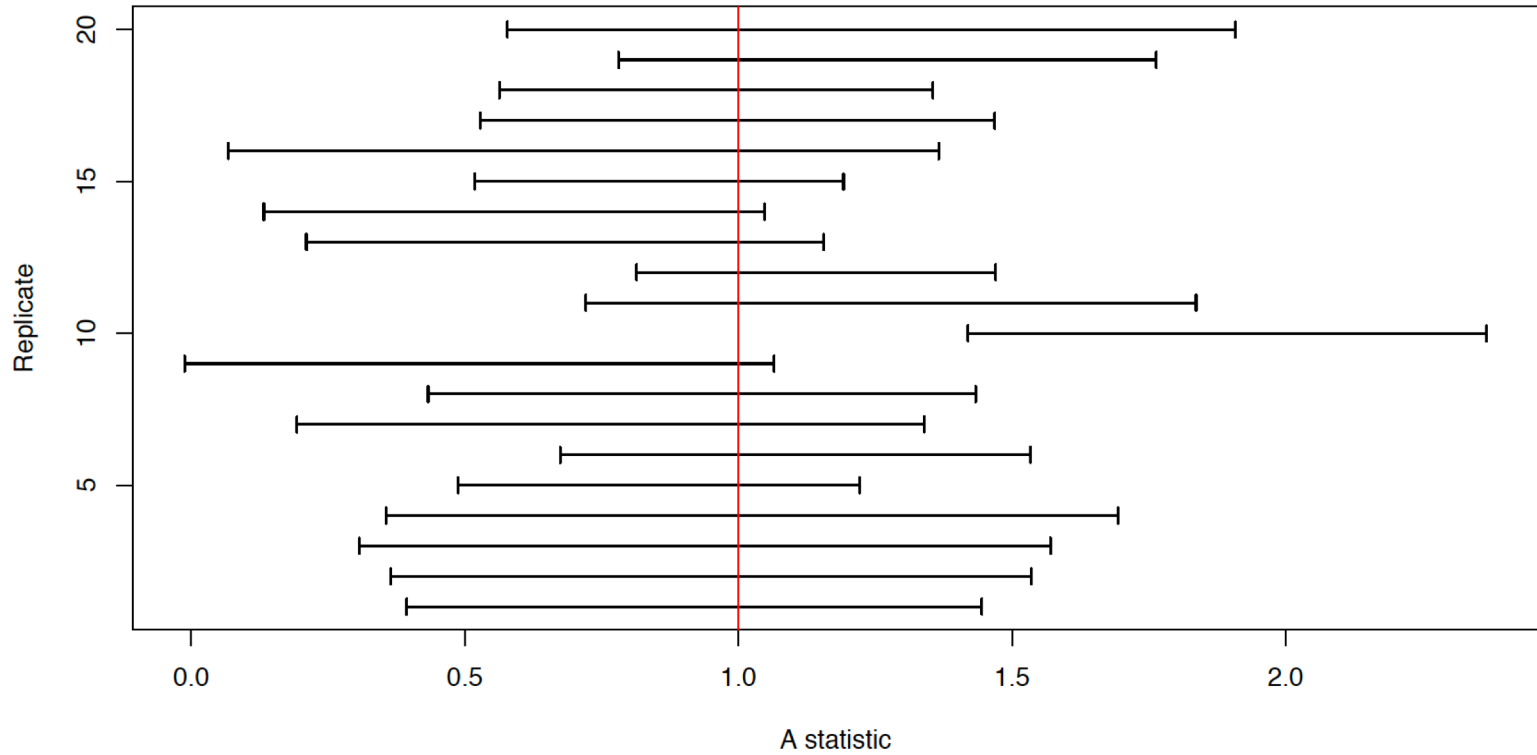
# Uncertainty

How can we quantify it?

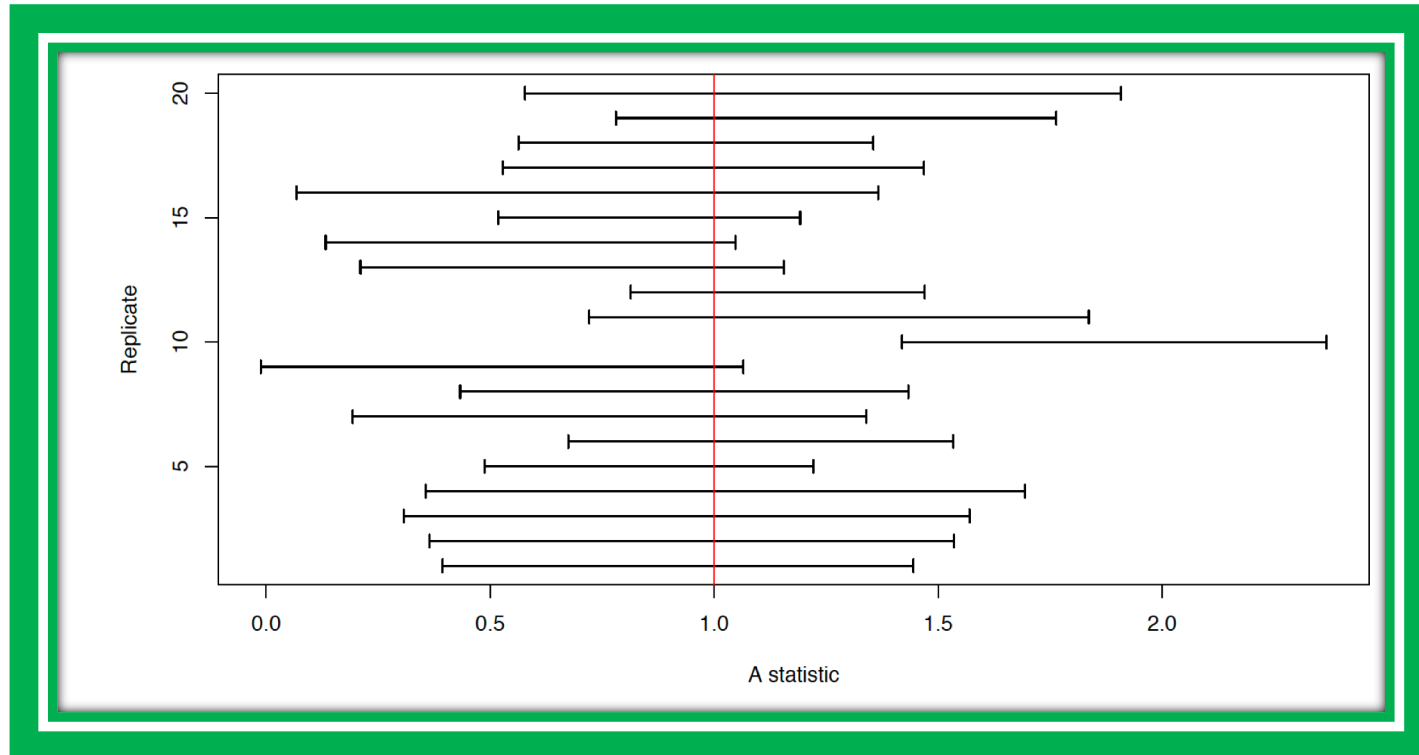
**Confidence intervals**



# Confidence intervals



# Confidence intervals

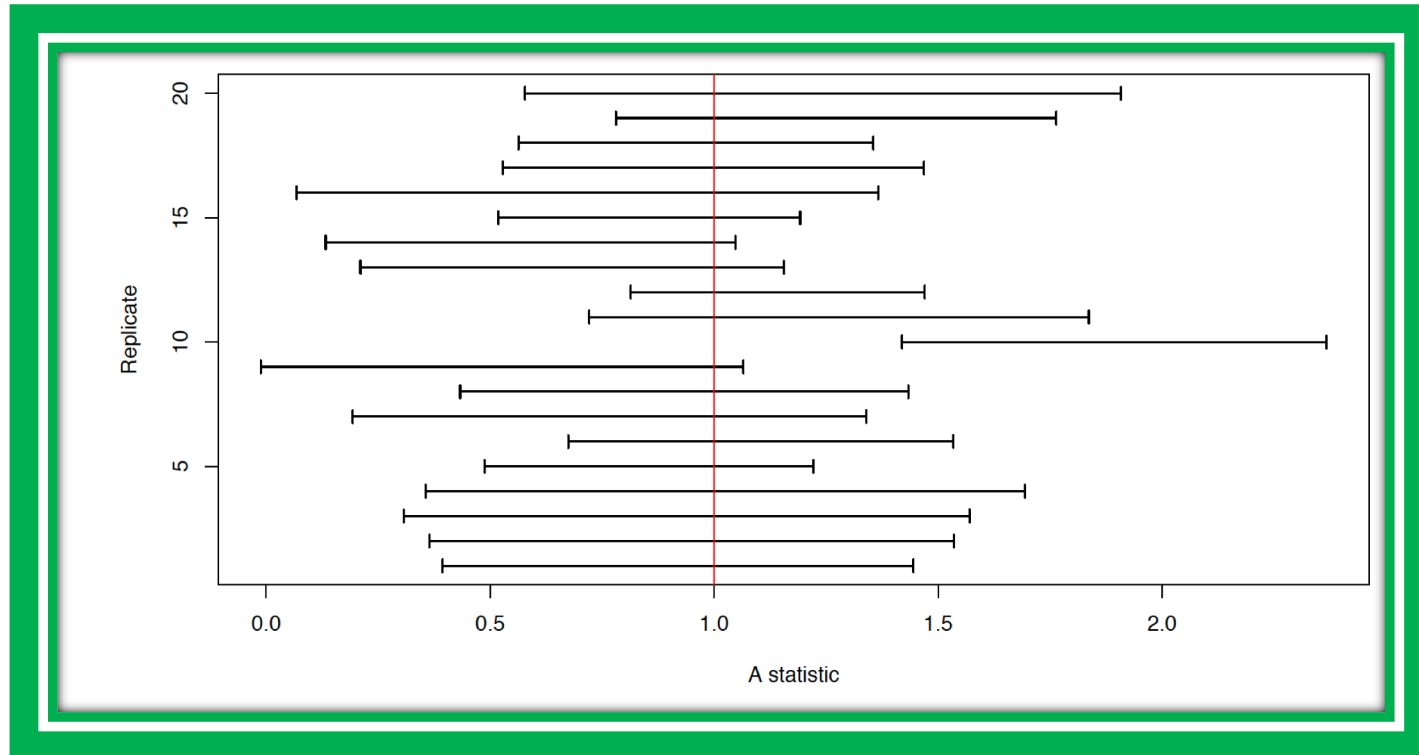


Confidence interval is used to indicate values for the true parameter that are more likely, given our data

IF you repeated your sampling many times and each time drew a confidence interval – 95% of the time (on average) the confidence interval would contain the true parameter value

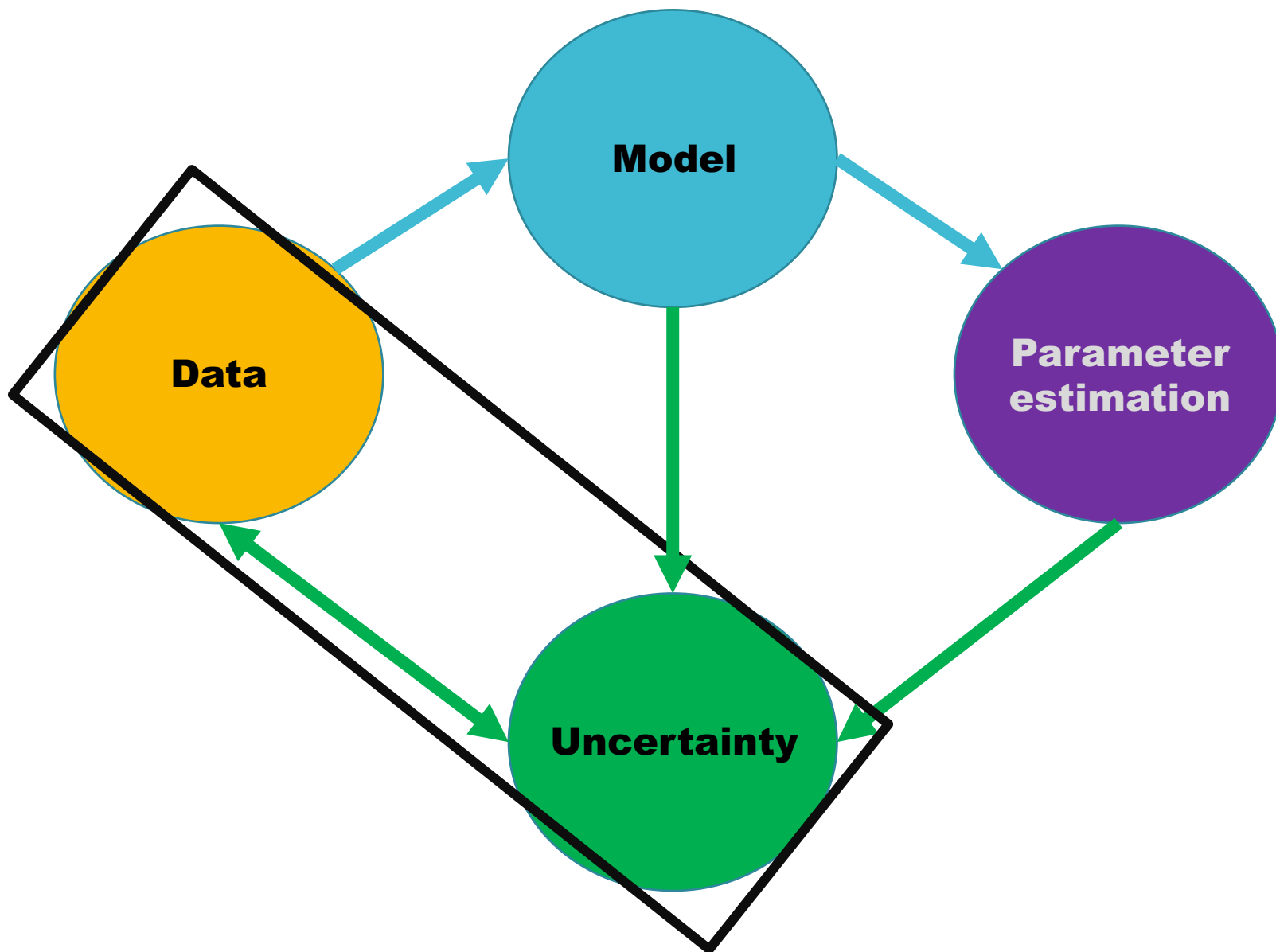


# Confidence intervals

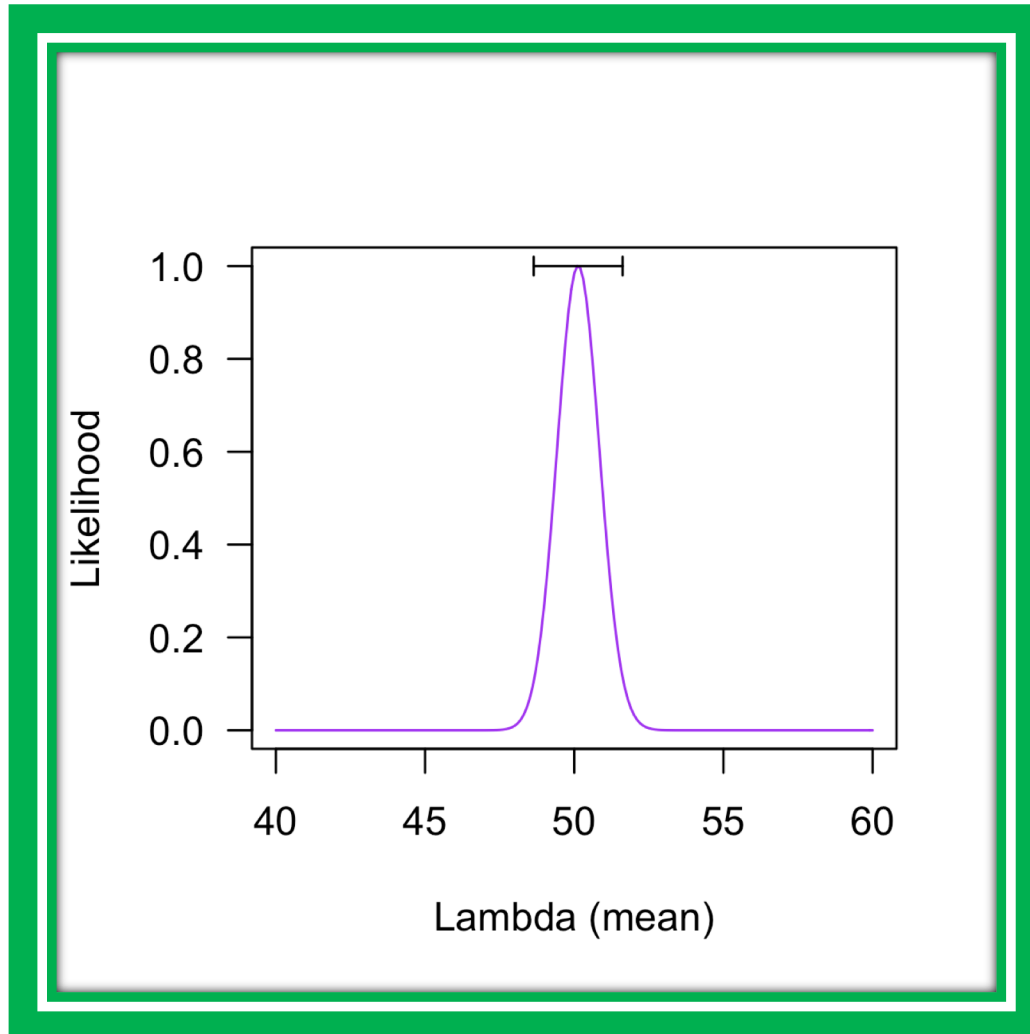


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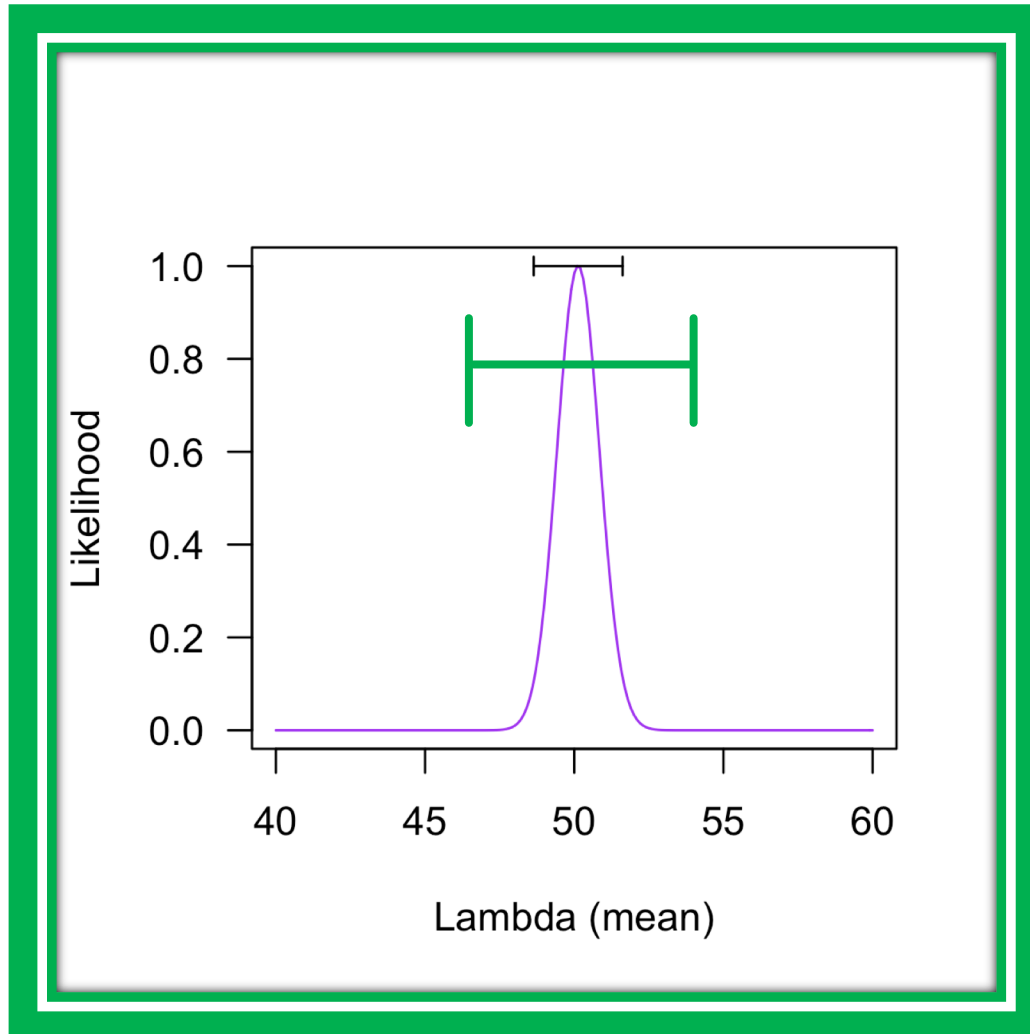
IF you repeated your sampling many times and each time drew a confidence interval – 95% of the time (on average) the confidence interval would contain the true parameter value **FREQUENTIST IDEA**



# Uncertainty



# Sample size and uncertainty



Less data =  
higher  
uncertainty in  
our estimates

# Interpretation

Meaning depends on the data and question

# Interpretation

**Meaning depends on the data and question:**

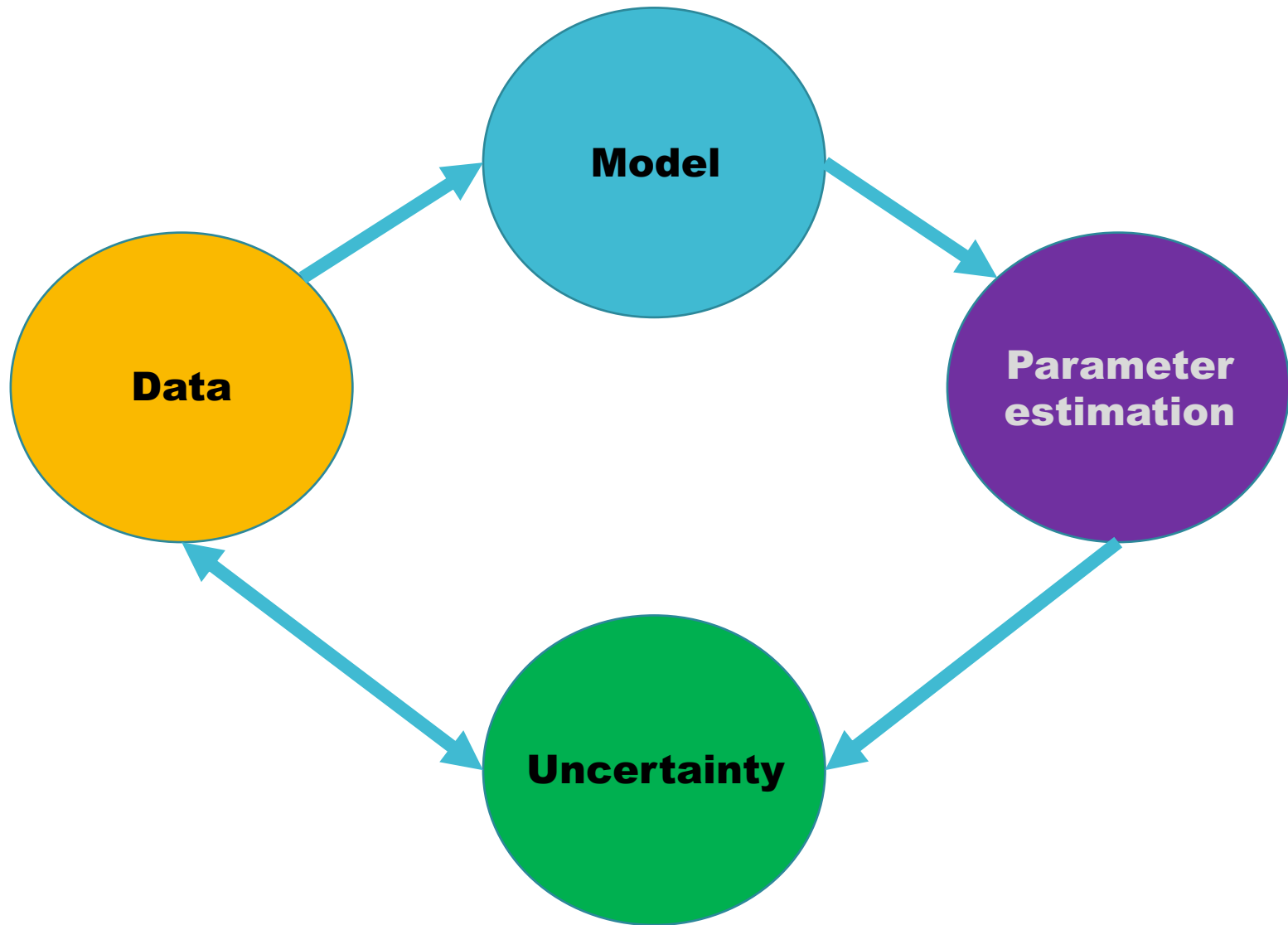
E.g. MLE of 50 lions and confidence interval of 49 to 52

= very good estimate in terms of uncertainty (only 3 lions variation)

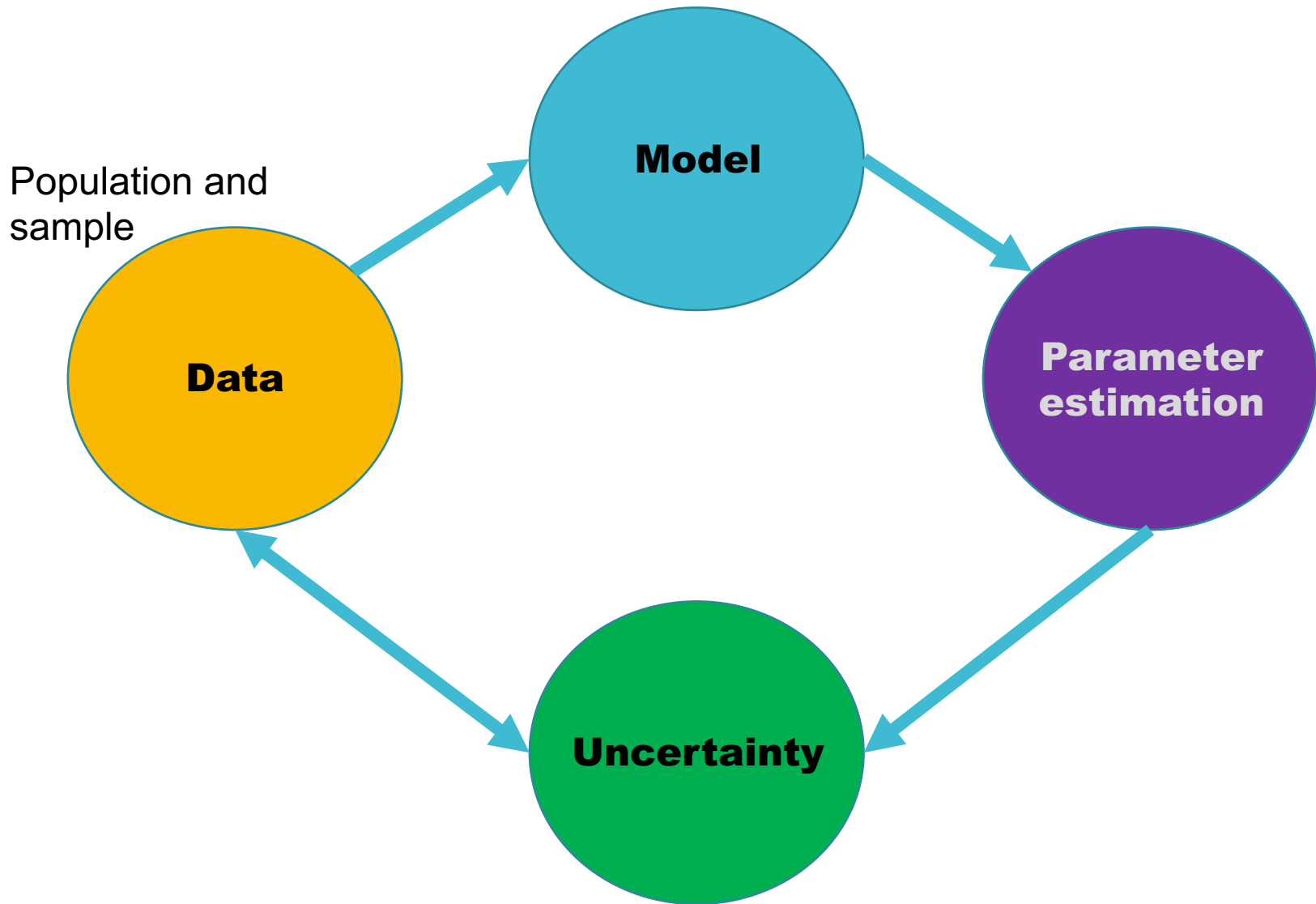
But

= useless if you need to be sure you capture all lions in an area,  
one lion still free could be too many

# Summary



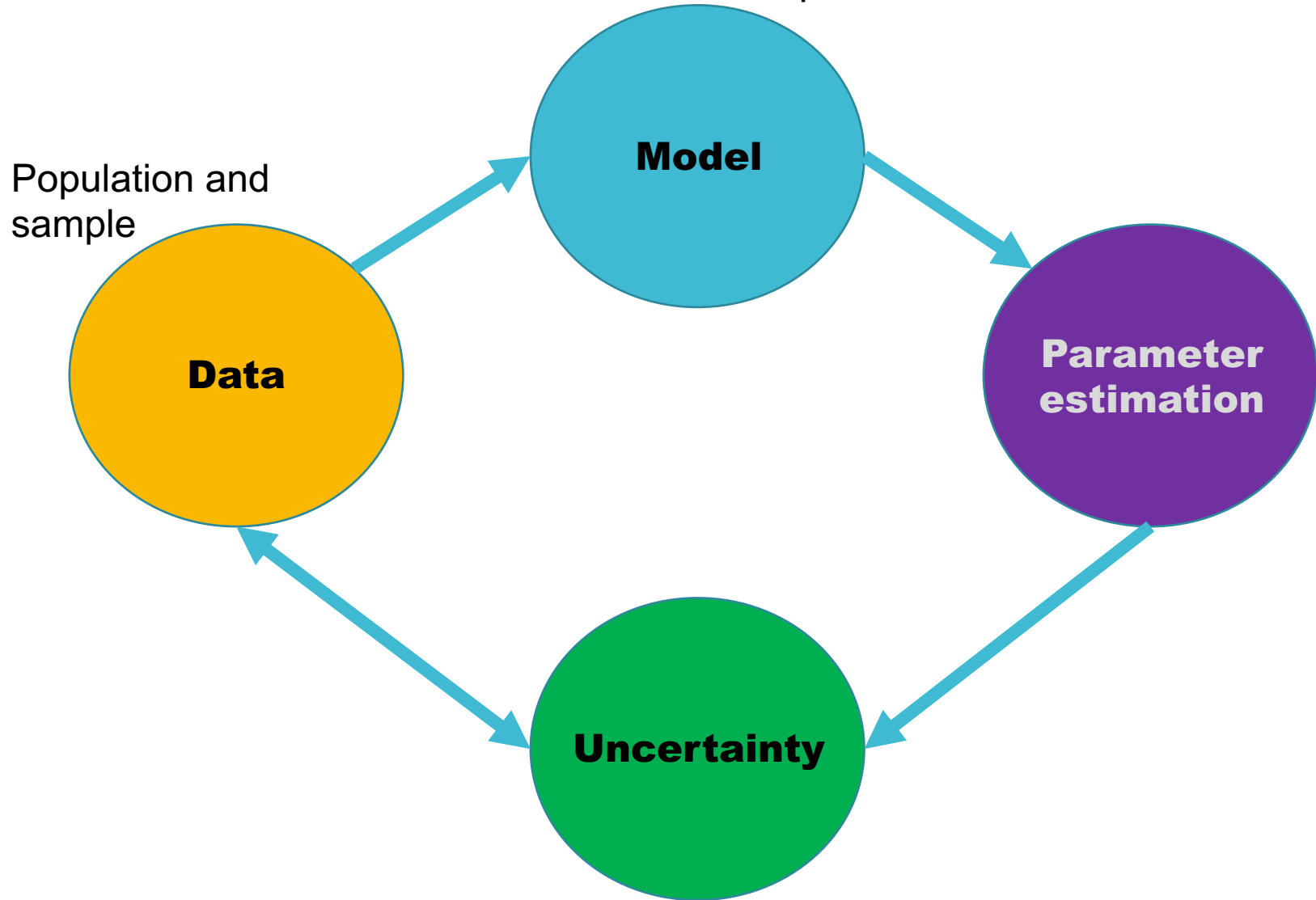
# Summary





# Summary

Match to data and question

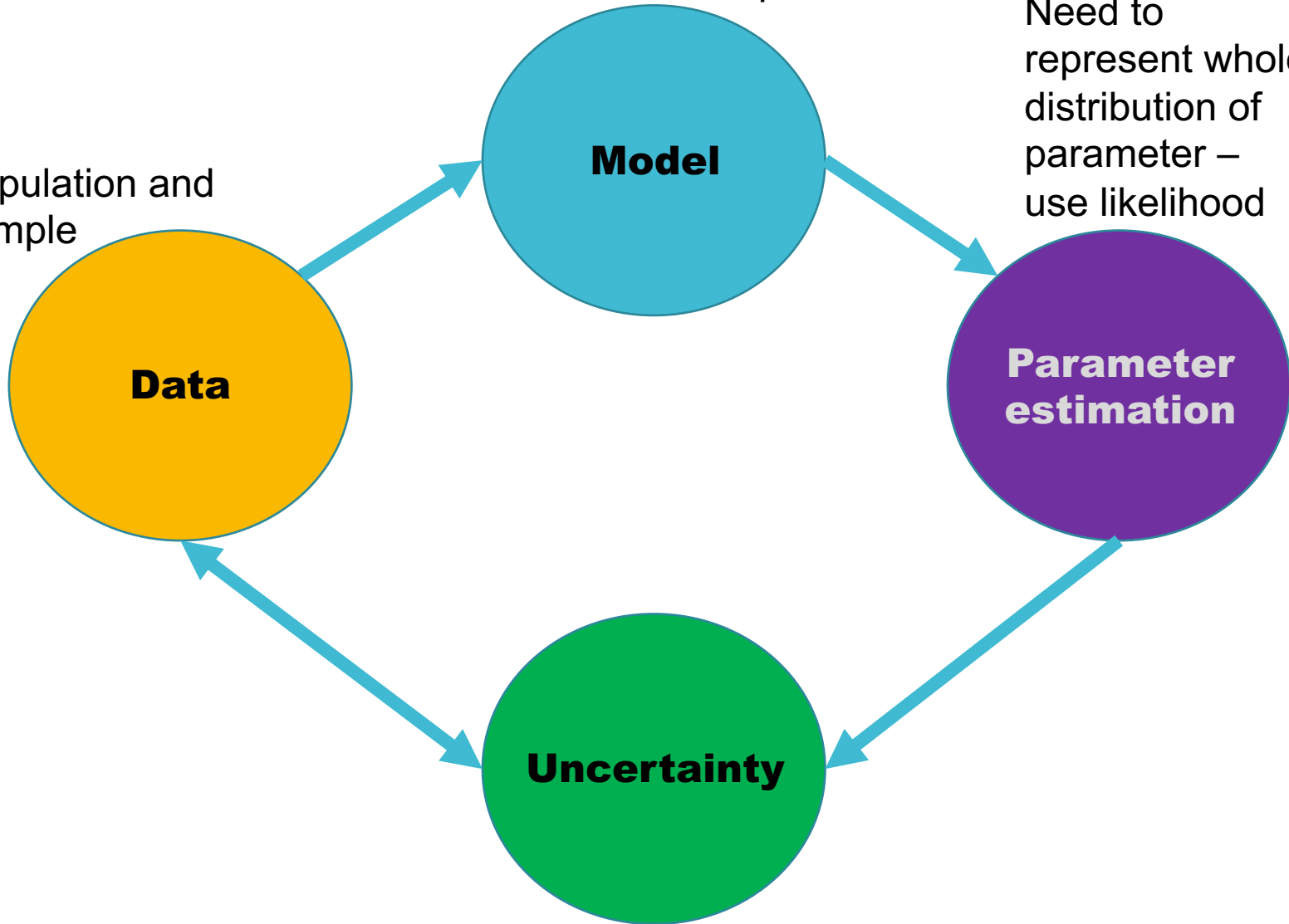


# Summary

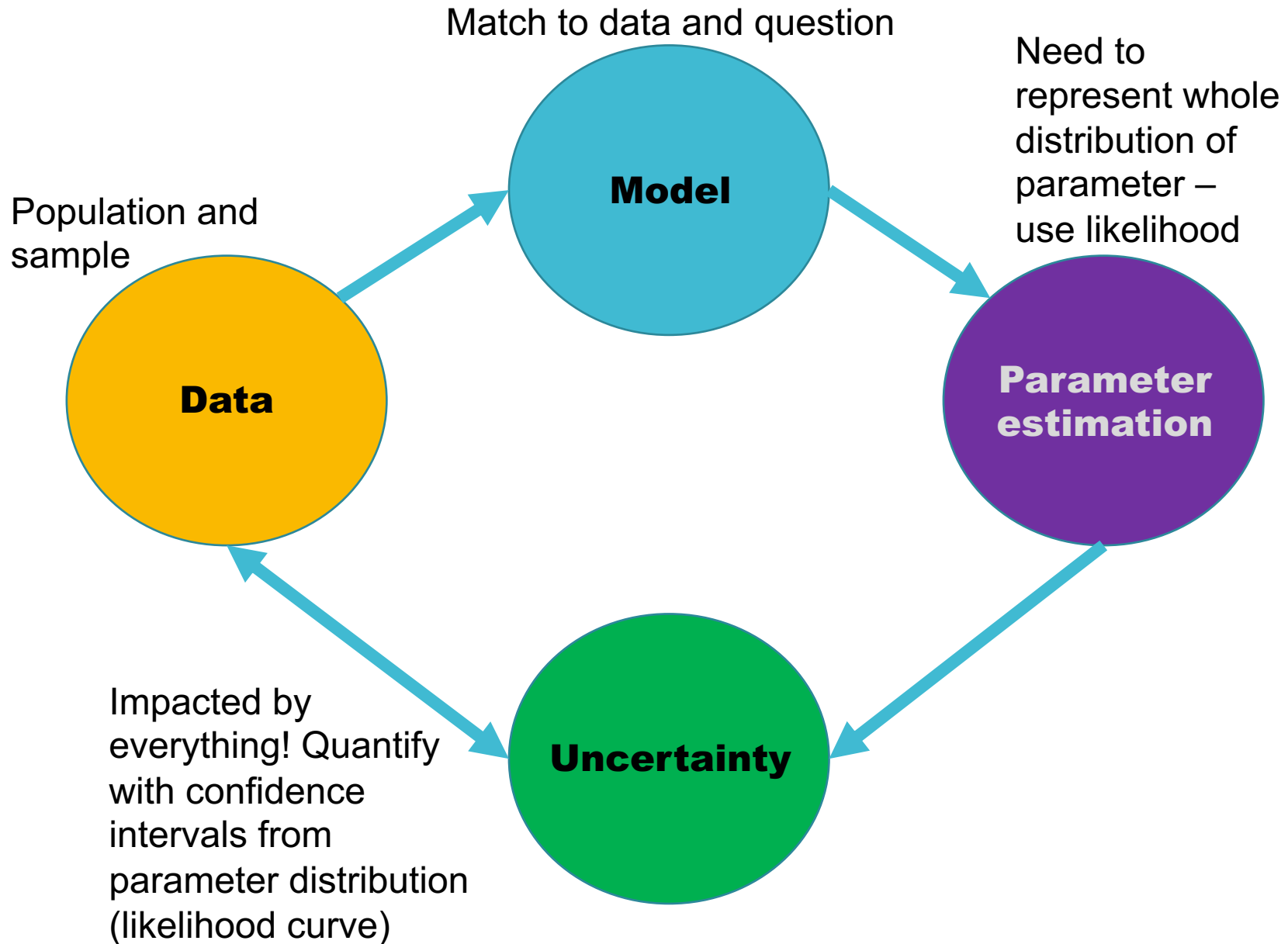
Match to data and question

Population and  
sample

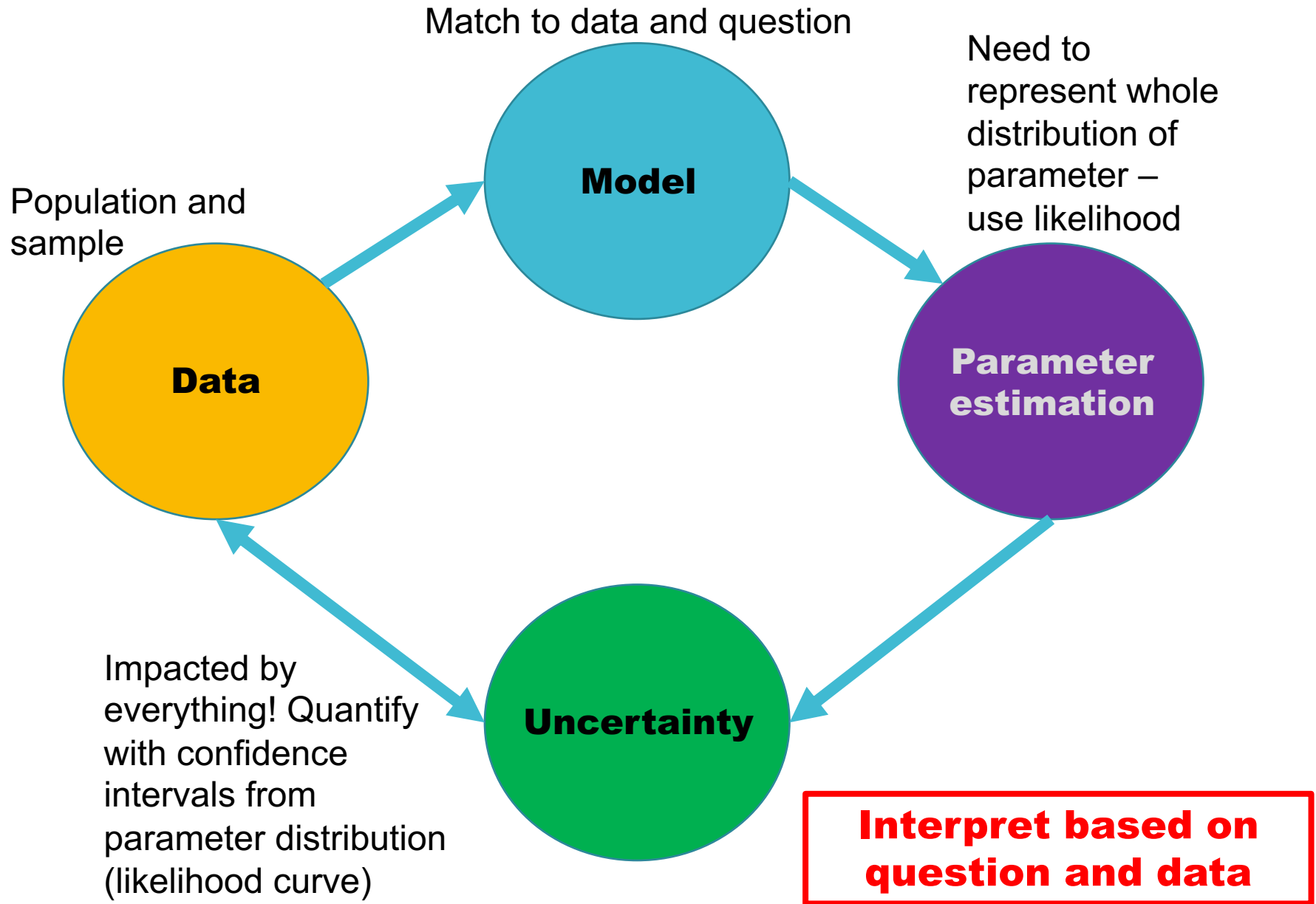
Need to  
represent whole  
distribution of  
parameter –  
use likelihood



# Summary



# Summary



# EXERCISE TASK

On your table write in your own words:

“What maximum likelihood estimation is and why we use it in statistical modelling”

**ASK any questions!!!**

