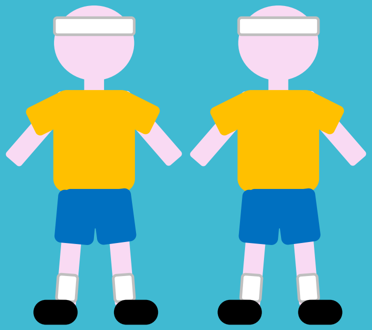
Linear regression: Part 2



Recap

What are linear models and linear regression?

How do we fit these models?

Using lm() in R

Lecture Outline

A bit more on fitting

Adding uncertainty

Interpretation of results

How do the results fit in the scientific process?

Lecture Outline

A bit more on fitting

- EX1: Fit regression for 100m times

Adding uncertainty

- EX2: Calculate confidence intervals

Interpretation of results

- EX3: Interpret the results

Prediction

- EX4: Prediction
- EX5: Discuss further steps/good models

A bit more on fitting

What the likelihood looks like

This is the log-likelihood for a linear regression:

$$l(y|x, \alpha, \beta, \sigma^2) = -\frac{n}{2}\log\sigma^2 - \sum_{i=1}^{n}\frac{(y_i - (\alpha + \beta x_i))^2}{2\sigma^2}$$

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The explanatory variable

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The response variable (our observed data)

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Our parameters

The explanatory variable

The response variable (our observed data)

The sample size

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This is the log-likelihood for a normal distribution:

$$l(y|\mu,\sigma^2) = -\frac{n}{2}\log\sigma^2 - \sum_{i=1}^{n}\frac{(y_i - \mu_i)^2}{2\sigma^2}$$

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Identical except:

$$\mu_i = (\alpha + \beta x_i)$$

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Identical except:
$\mu_i = (\alpha + \beta x_i)$ to get the mean for the normal distribution we use the linear equation

What the likelihood looks like

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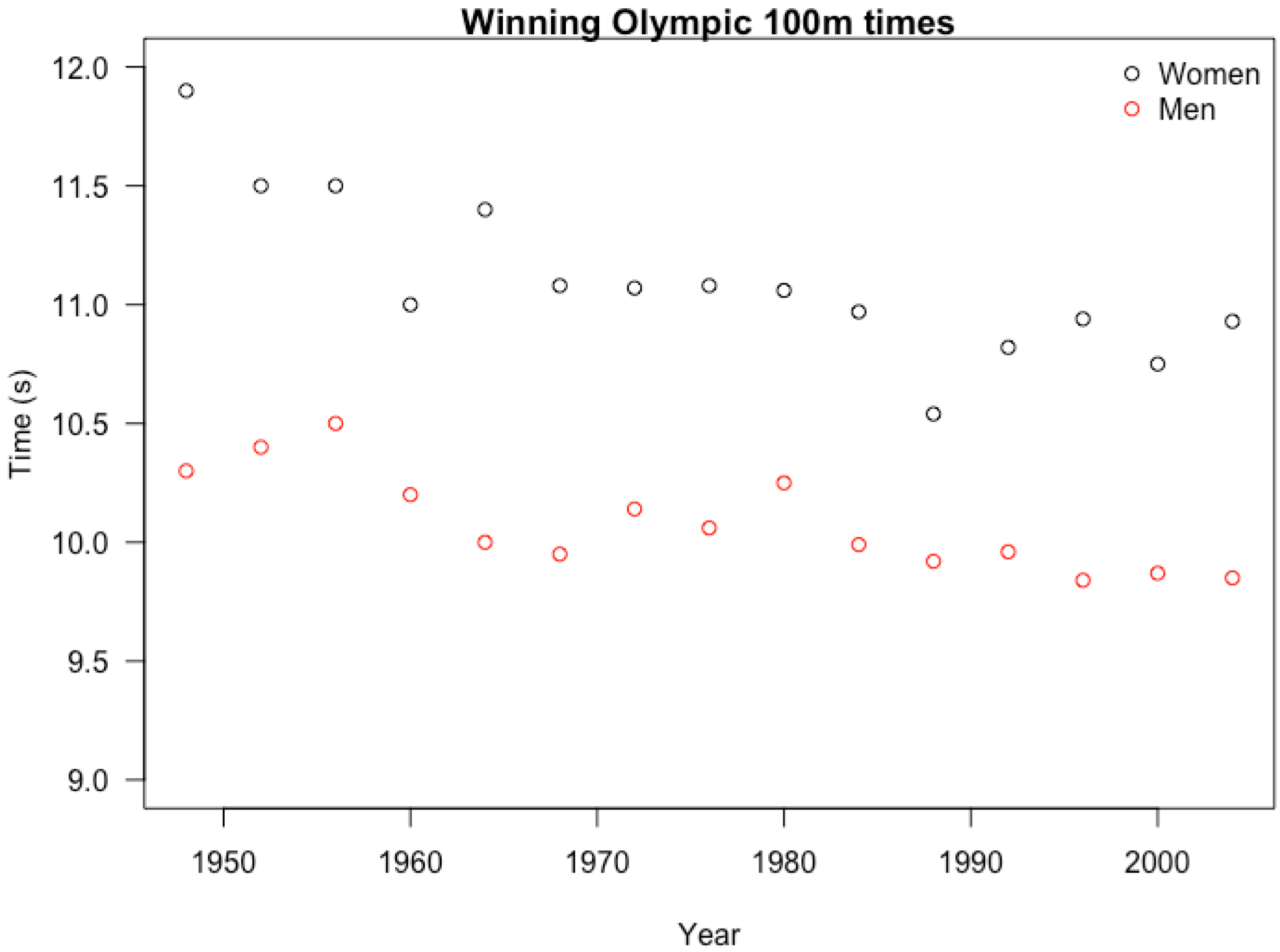
This part is the same as summing the squares (yesterday)

Data for today



Reminder! Fitting a linear regression in R

Arguments of lm():

lm(formula, data)

formula = Y ~ X
data = your data

Y is the response variable
X is the explanatory variable

Exercise 1: Fit regression to 100m times

Part E of exercise module.

Some groups will run a regression on the women’s times, the others will do one on the men’s times (ONLY DO ONE)

Adding uncertainty/ confidence

Exercise 2: Adding confidence

Part F

Some theory and practice

Interpretation of results

Exercise 3: Interpret your results.

Part G

Practice interpreting the results

Exercise 3: Present results

5 minutes to update your results

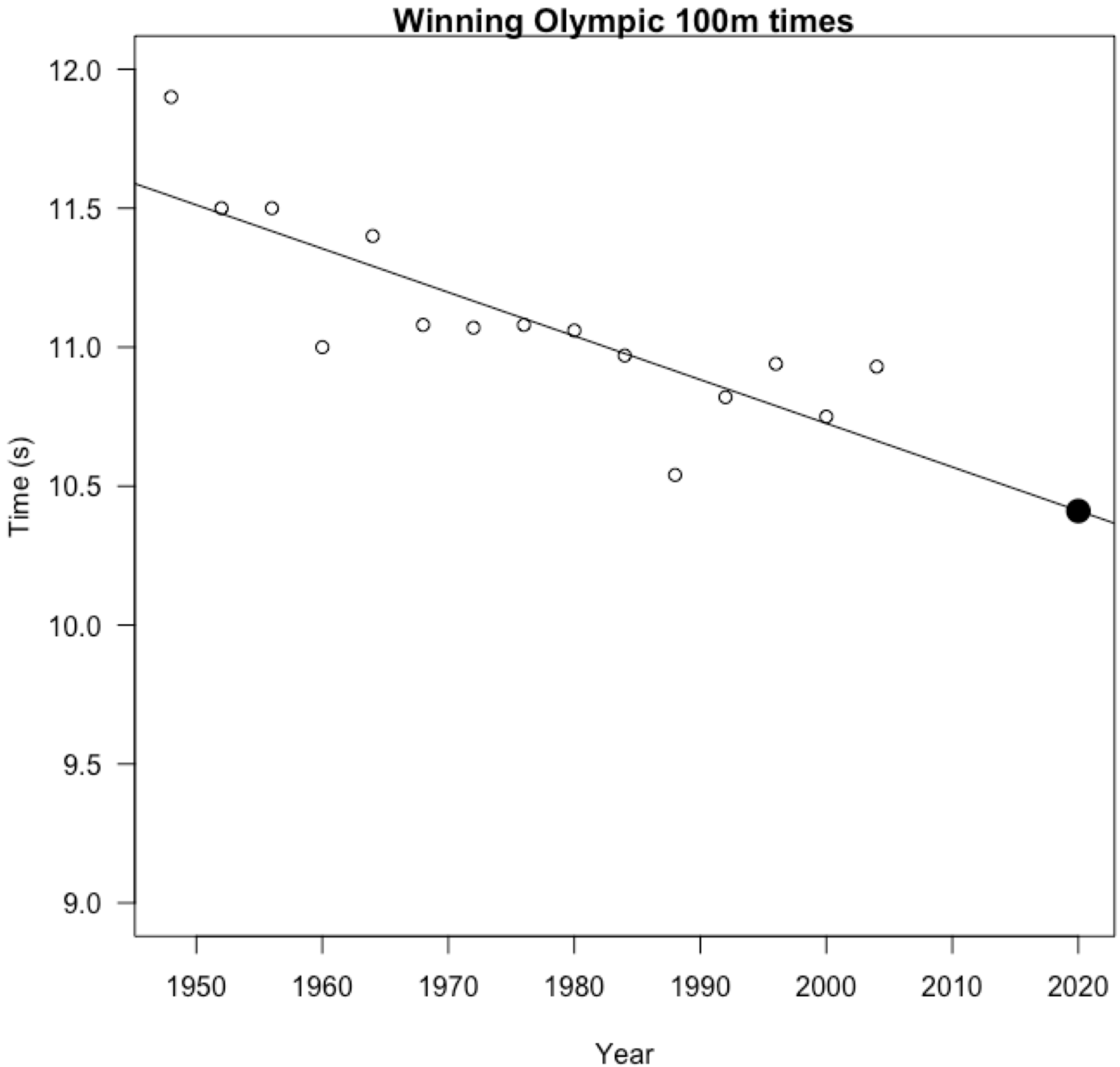
Turn to same row on opposite side and tell them your result

Is it different for men and women?

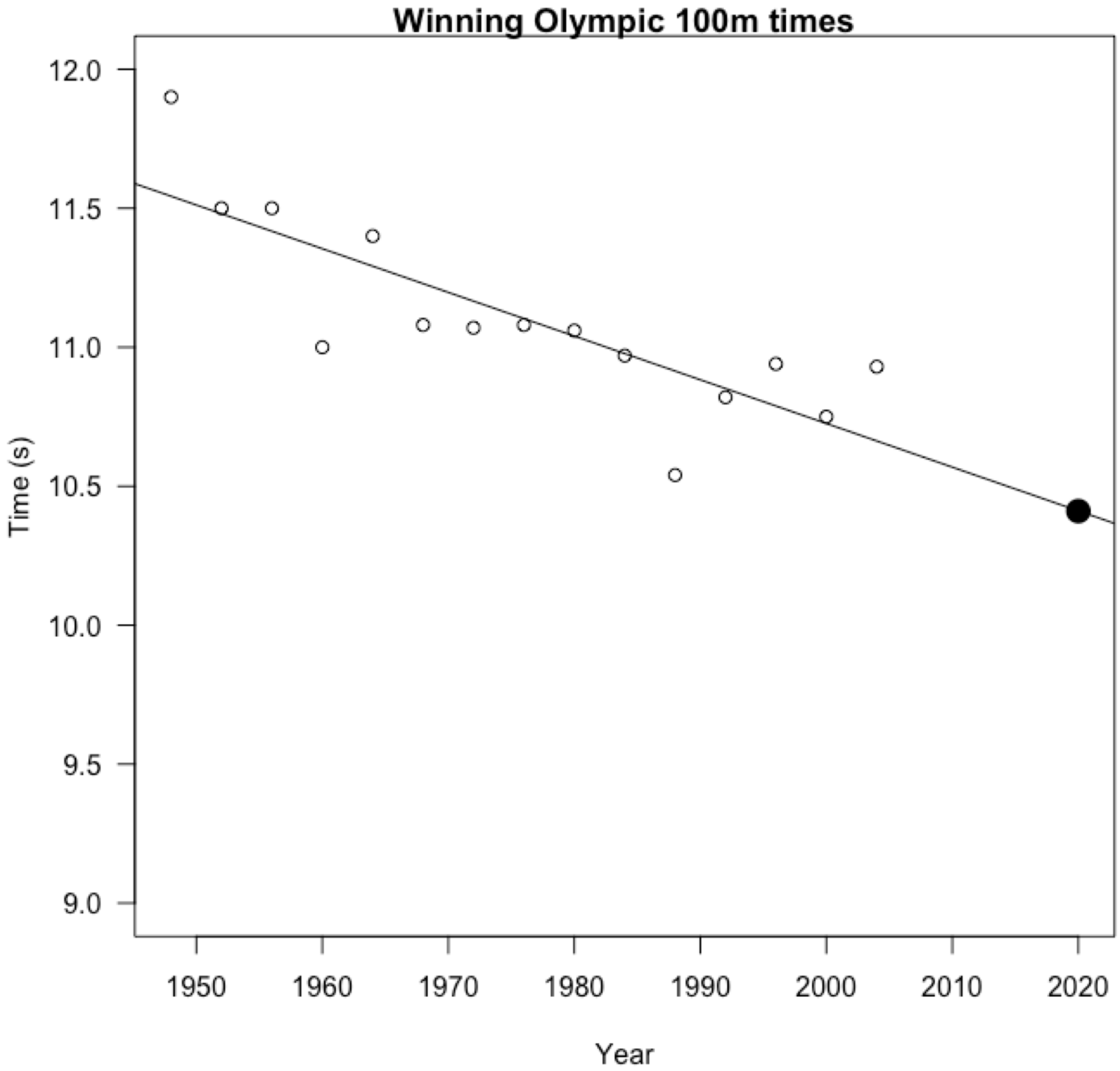
Exercise 4: Prediction

Finish part G

Uncertainty in prediction



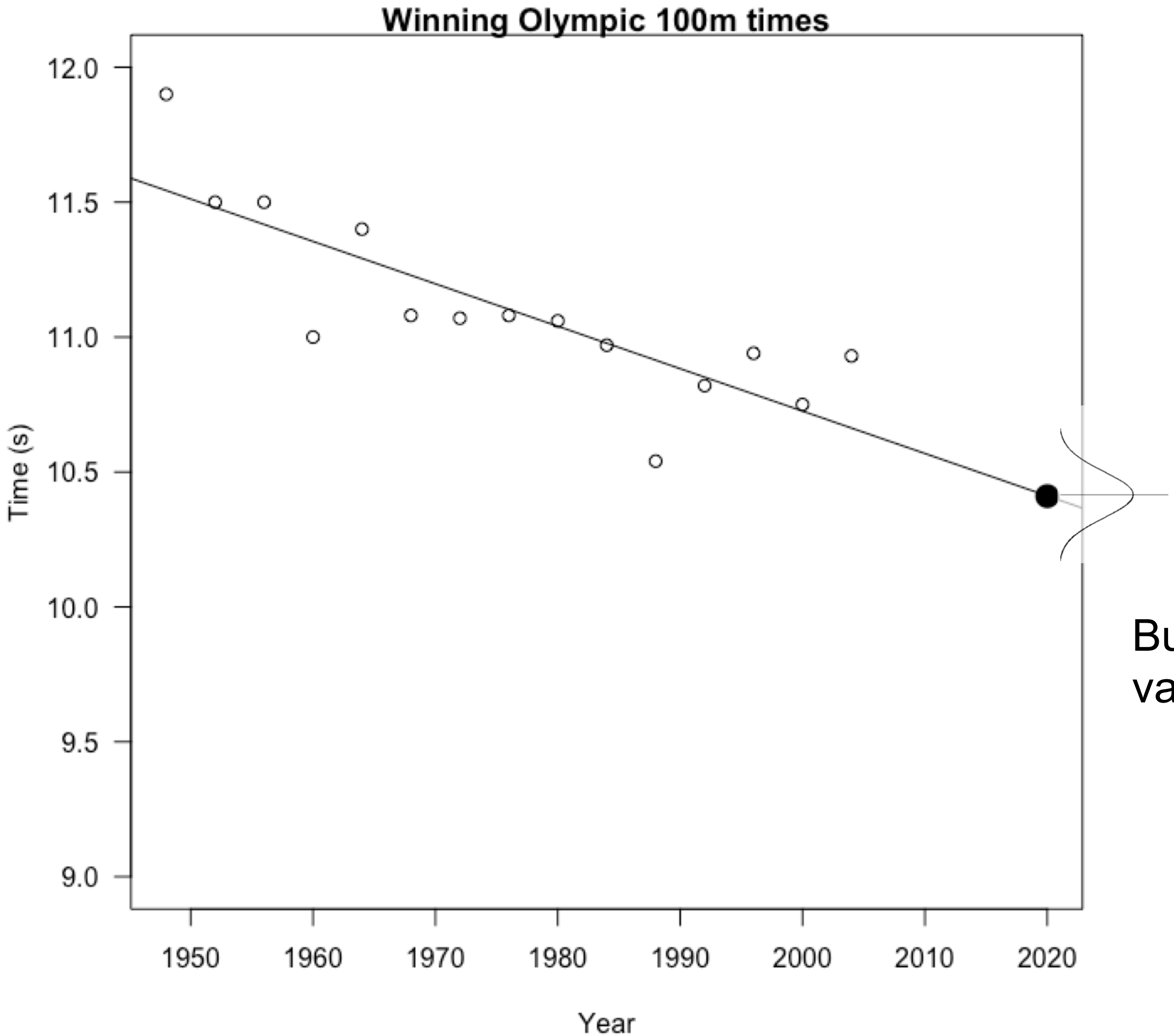
Uncertainty in prediction



X = 2020

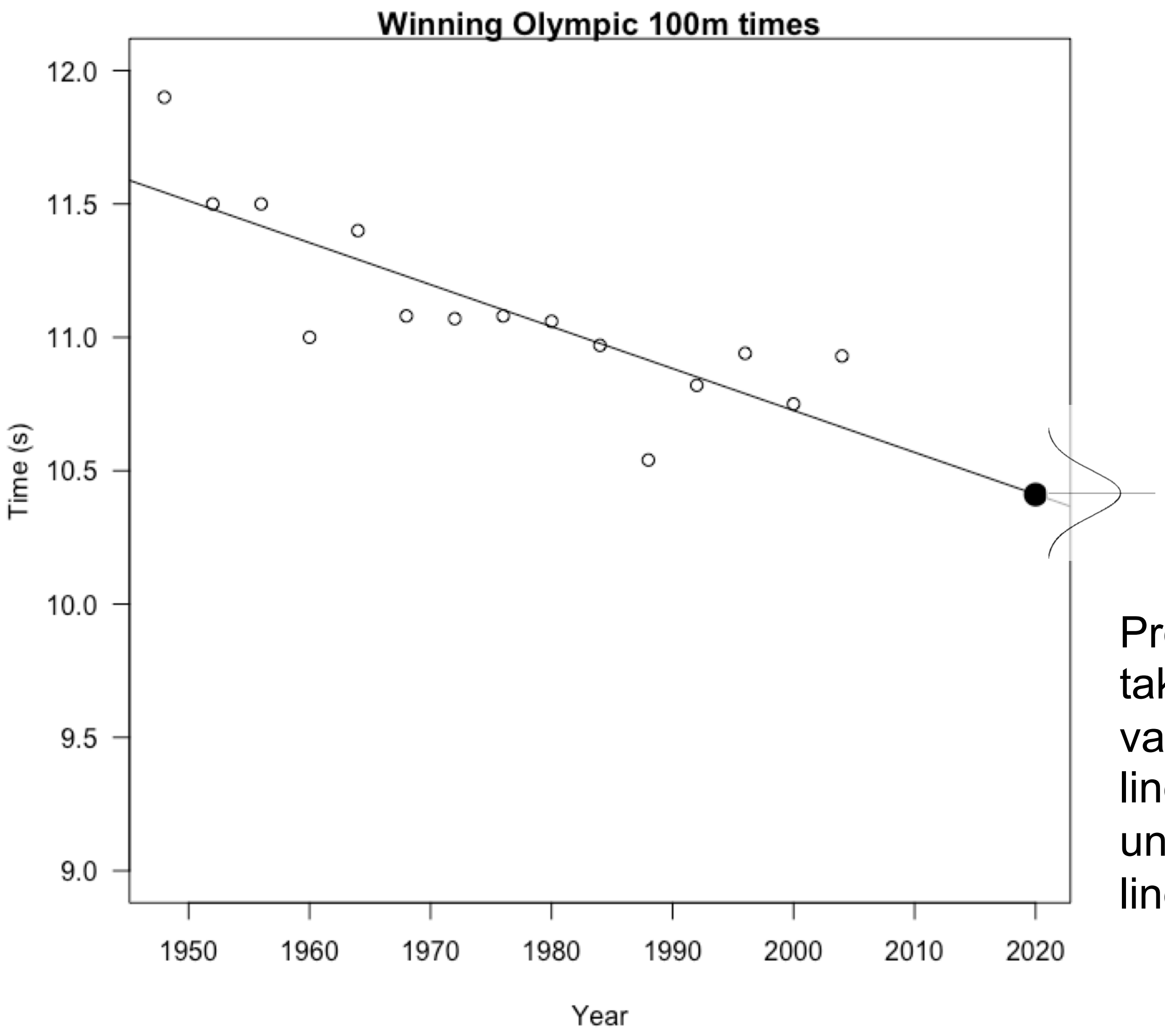
$\hat{Y}$ = 10.41 seconds

Uncertainty in prediction



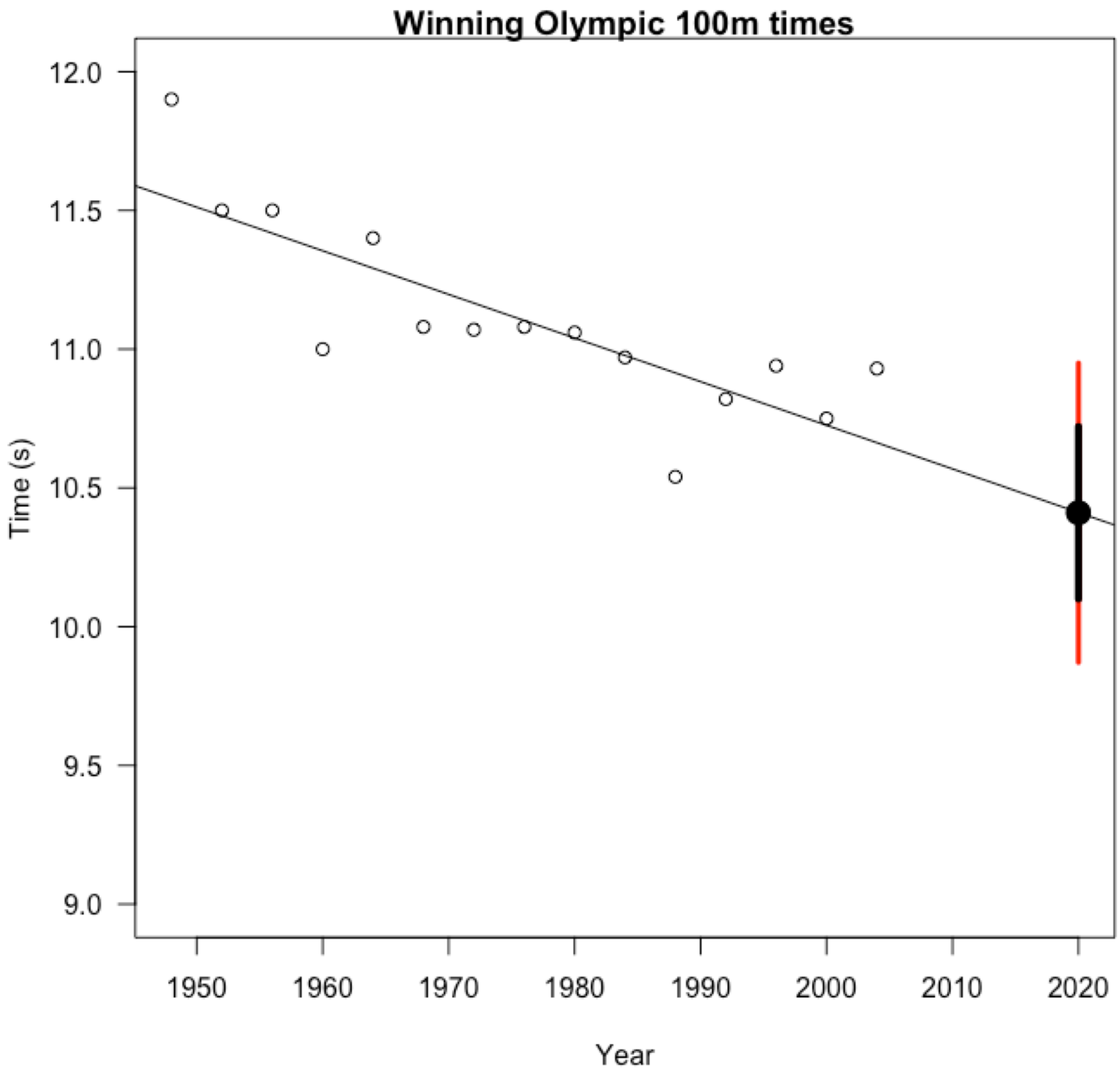
But what about variation???

Uncertainty in prediction

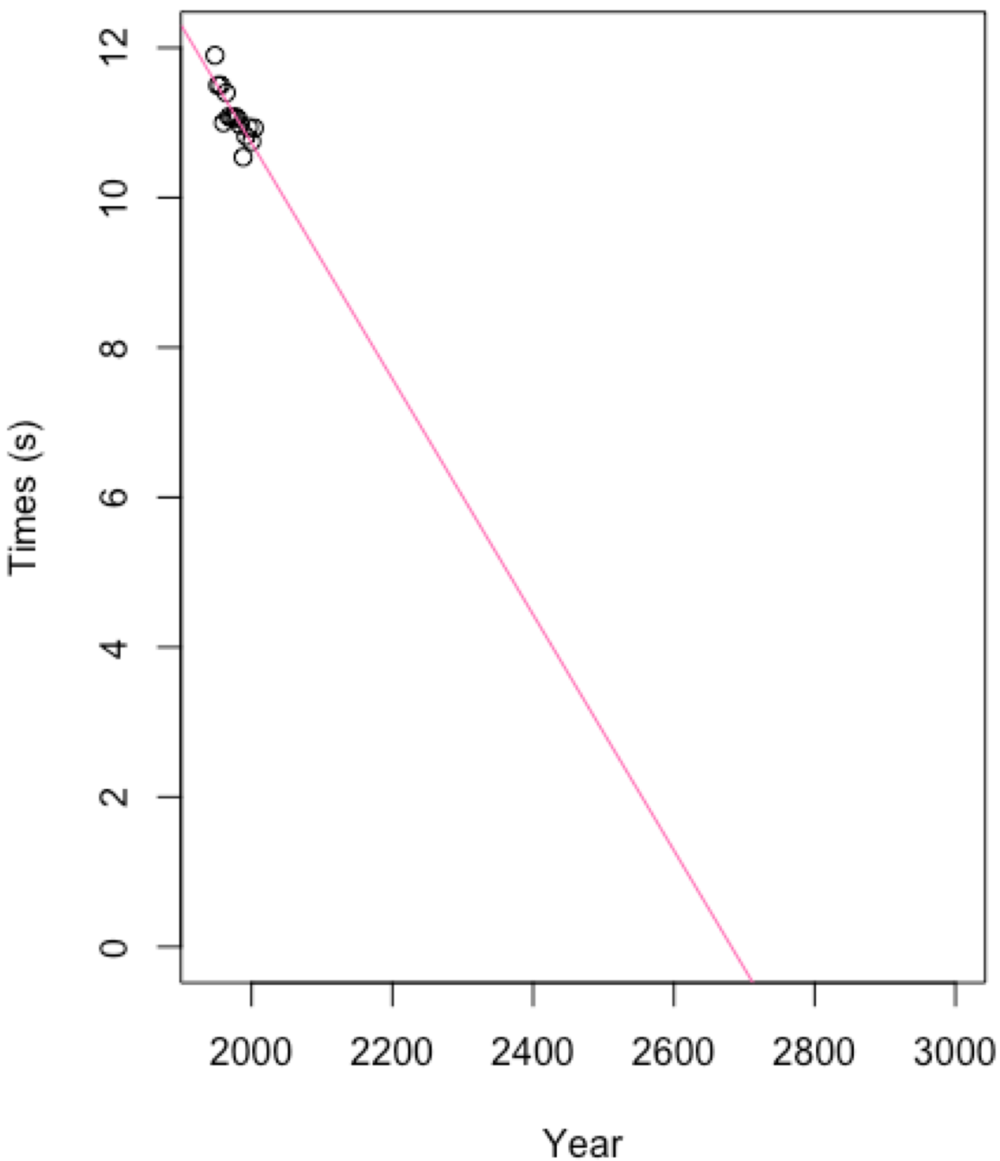


Prediction interval takes into account variation around the line as well as uncertainty in the line itself!

Uncertainty in prediction



95% prediction interval for women in 2020 is between 9.87 and 10.94 seconds



Be careful with prediction

Exercise 5: Further directions

Part H

Exercise 5: Further directions

Feedback on further directions

Lecture Summary

A bit more on fitting

Adding uncertainty

Interpretation of results

Prediction

Lecture Summary

A bit more on fitting
Tried for a real example

Adding uncertainty

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We add uncertainty to represent taking a sample many times

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Interpretation of results

We can translate α β into change in Y with X (back into biological units) – make conclusion about relationship

Prediction

Lecture Summary

A bit more on fitting
Tried for a real example

Adding uncertainty
We add uncertainty to represent taking a sample many times

Interpretation of results
We can translate α β into change in Y with X (back into biological units) – make conclusion about relationship

Prediction
Can be useful but also need to be careful of going too far outside of your data

Give us feedback