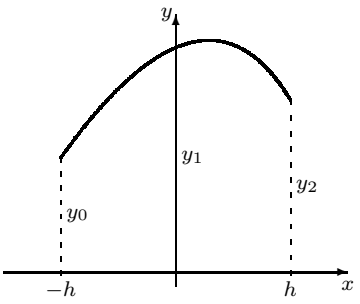
Grunnlaget for Simpsons metode

(fra forelesningen mandag 19. september kl. 0815–1000)

$y = Ax^2 + Bx + C$



Påstand:

$$\int_{-h}^{h} \Big(Ax^2 + Bx + C \Big) dx = \frac{h}{3} \Big(y_0 + 4y_1 + y_2 \Big)$$

Bevis:

$$\left.\begin{array}{ll} x = -h: & y_0 = Ah^2 - Bh + C \\ x = \ \ 0: & y_1 = C \\ x = \ \ h: & y_2 = Ah^2 + Bh + C \end{array}\right\} \quad \Longrightarrow \quad \begin{array}{ll} C = y_1 & (1) \\ y_0 + y_2 = 2Ah^2 + 2C & \\ 2Ah^2 = y_0 + y_2 - 2y_1 & (2) \end{array}$$

$$\underline{\underline{\int_{-h}^{h} \Big(Ax^2 + Bx + C \Big) dx}} = \left[\frac{A}{3}x^3 + \frac{B}{2}x^2 + Cx \right]_{-h}^{h}$$

$$= \frac{2A}{3}h^3 + 2Ch$$

$$= \frac{h}{3} \Big(2Ah^2 + 6C \Big) \qquad \big(\text{setter inn (1) og (2)}\big)$$

$$= \frac{h}{3} \Big((y_0 + y_2 \ - 2y_1) + 6y_1 \Big) = \underline{\underline{\frac{h}{3} \Big(y_0 + 4y_1 + y_2 \Big)}}$$