THEOREM 1 The Natural Growth Equation
The solution of the initial value problem

$$\frac{dx}{dt} = kx, \qquad x(0) = x_0$$

is

$$x(t) = x_0 e^{kt}.$$

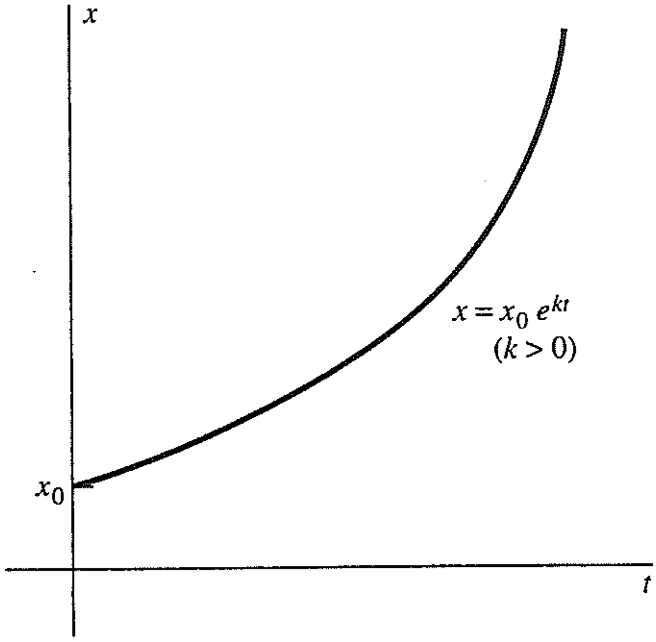


FIGURE 9.1.2 Solution of the exponential growth equation for $k > 0$.

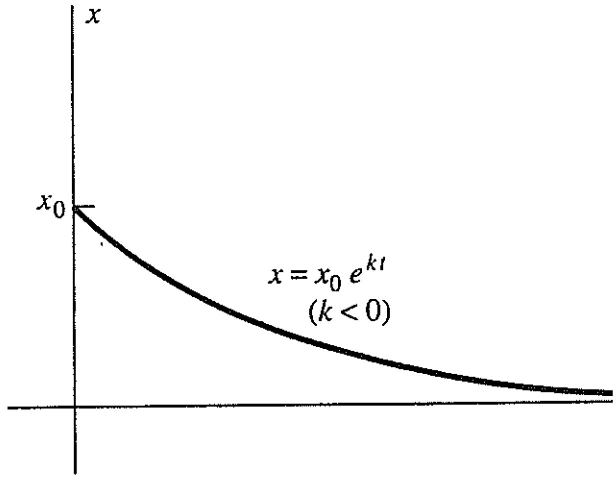


FIGURE 9.1.3 Solution of the exponential growth equation—now actually a *decay* equation—for the case $k < 0$.

EXAMPLE 4 A specimen of charcoal found at Stonehenge contains 63% as much ^{14}C as a sample of present-day charcoal. What is the age of the sample?

EXAMPLE 8 The water in a draining cylindrical tank is 10 ft deep at noon. At 1:00 P.M. it is 5 ft deep. When will the tank be empty?

Tenk deg at du er eier av et luksushotell med et lekkert utendørs badeanlegg. Vannet i bassenget holdes jevnt på 28°. Dessverre bryter bassengets varmeanlegg sammen 24 timer før badeforeningen "De glade badere" har bebudet sitt komme for å avholde sin årlige badefestival. Skal du avlyse evenementet (og derved tape inntekt), eller vil badetemperaturen fremdeles være akseptabel etter et døgn? Du trekker naturligvis et plastdekke over bassenget for å minske fordampningen, men lufttemperaturen er bare 12°, og værmeldingen har lovet at der vil den holde seg.

Newtons avkjølingslov sier at avkjølingshastigheten er proporsjonal med temperaturforskjellen

Method: Solution of First-Order Linear Equations

1. Begin by calculating the integrating factor $\rho(x) = e^{\int P(x)\,dx}$.
2. Then multiply both sides of the differential equation by $\rho(x)$.
3. Next, recognize the left-hand side of the resulting equation as the derivative of a product:

$$D_x[\rho(x)y(x)] = \rho(x)Q(x).$$

4. Finally, integrate this equation to obtain

$$\rho(x)y(x) = \int \rho(x)Q(x)\,dx + C,$$

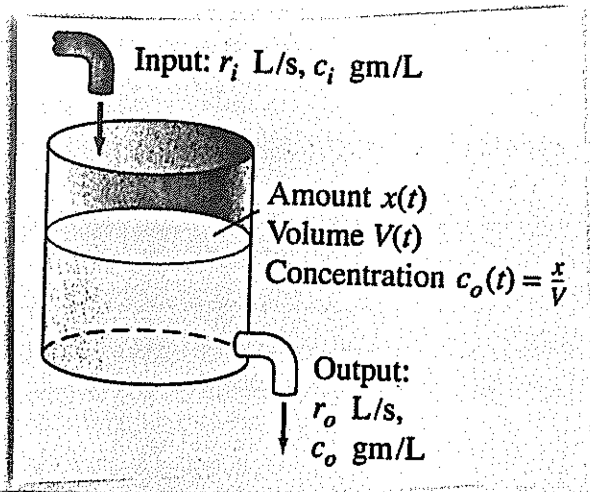
then solve for y to obtain the general solution of the original differential equation.

THEOREM 1 The Linear First-Order Equation

If the functions $P(x)$ and $Q(x)$ are continuous on the open interval I containing the point x_0, then the initial value problem

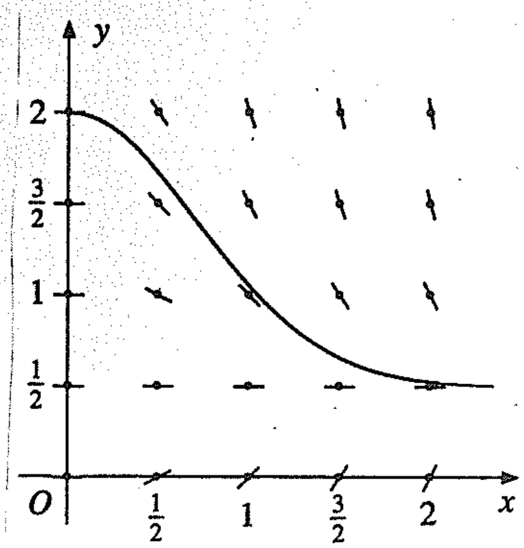
$$\frac{dy}{dx} + P(x)y = Q(x), \qquad y(x_0) = y_0 \tag{10}$$

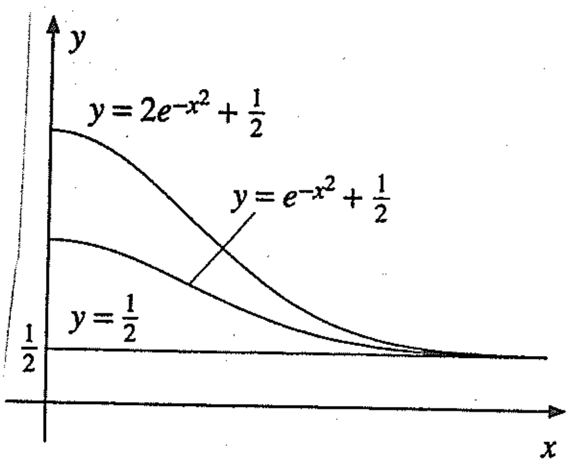
has a unique solution $y(x)$ on I, given by the formula in (5) with an appropriate value of C.



EXAMPLE 6 A 120-gal tank initially contains 90 lb of salt dissolved in 90 gal of water. Brine containing 2 lb/gal of salt flows into the tank at the rate of 4 gal/min, and the well-stirred mixture flows out of the tank at the rate of 3 gal/min. How much salt does the tank contain when it is full?

y_0 \ x_0	0	1/2	1	3/2	2
0	0	1/2	1	3/2	2
1/2	0	0	0	0	0
1	0	−1/2	−1	−3/2	−2
3/2	0	−1	−2	−3	−4
2	0	−3/2	−3	−9/2	−6





EXAMPLE 1 Construct a slope field for the differential equation $dy/dx = x - y$ and use it to sketch a solution curve that passes through the point $(-4, 4)$.

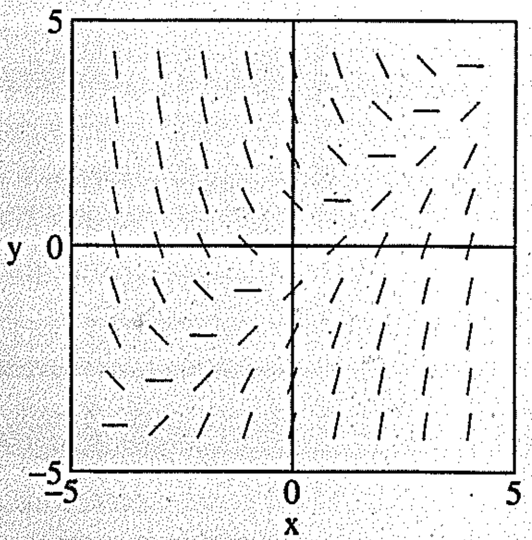


FIGURE 9.2.3 Slope field for $y' = x - y$ corresponding to the table of slopes in Fig. 8.2.2.

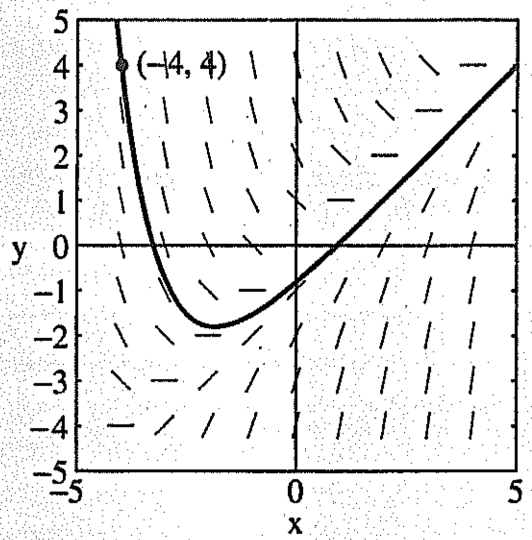
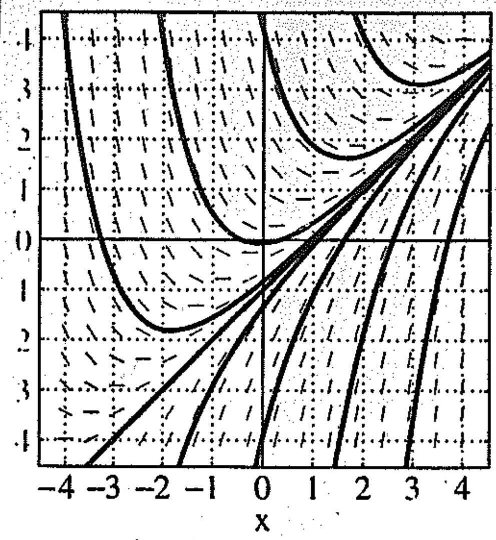
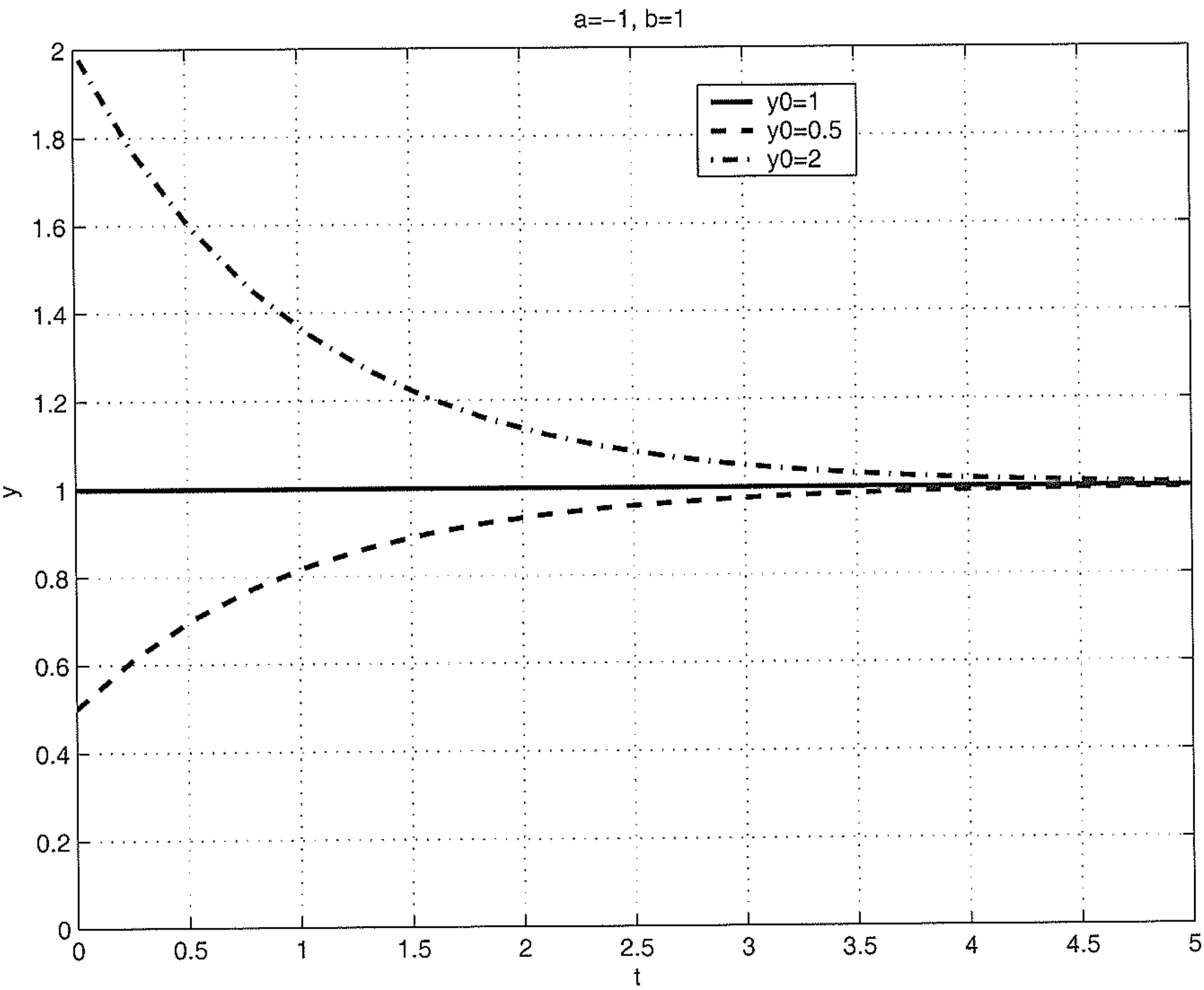
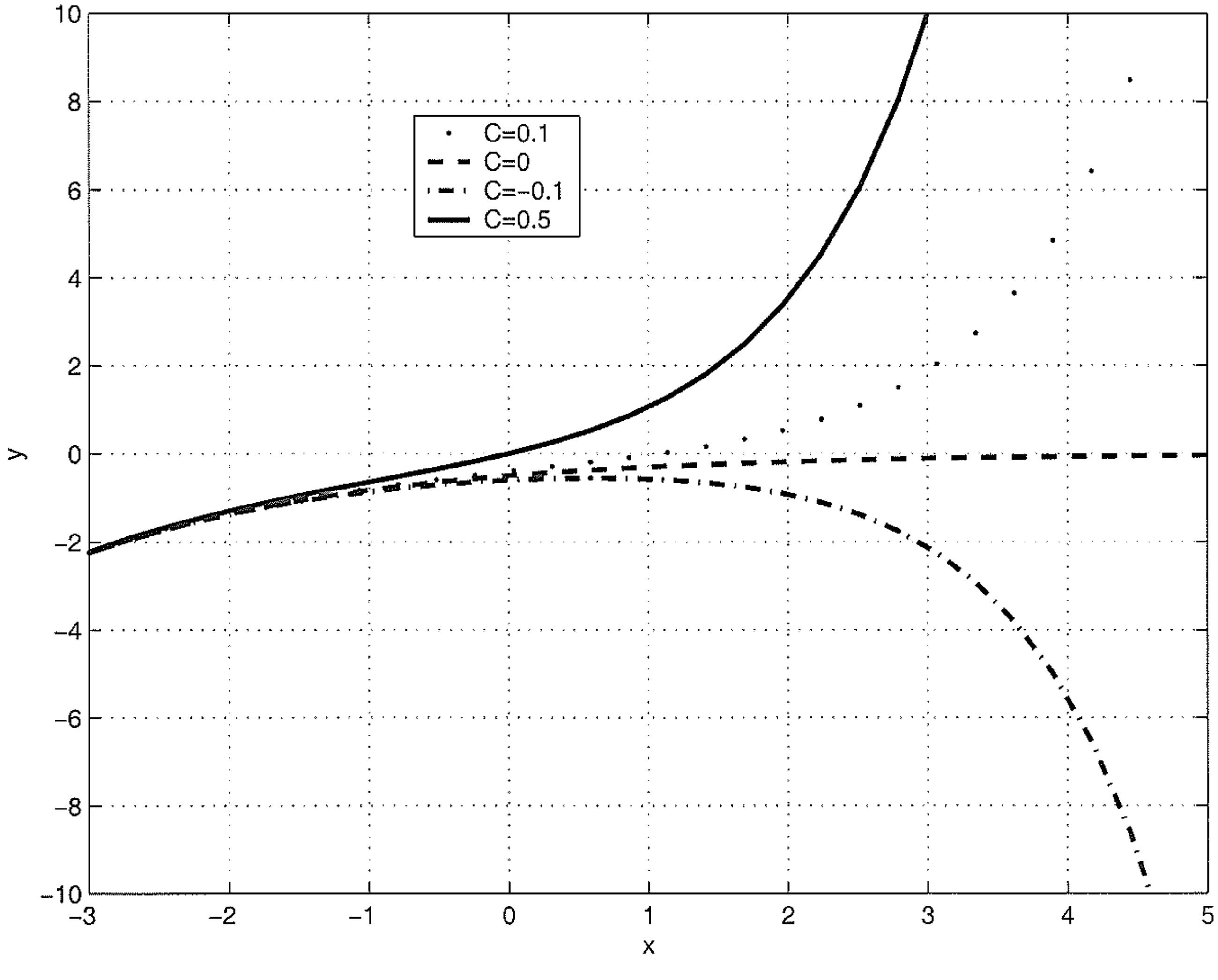


FIGURE 9.2.4 The solution curve through $(-4, 4)$.

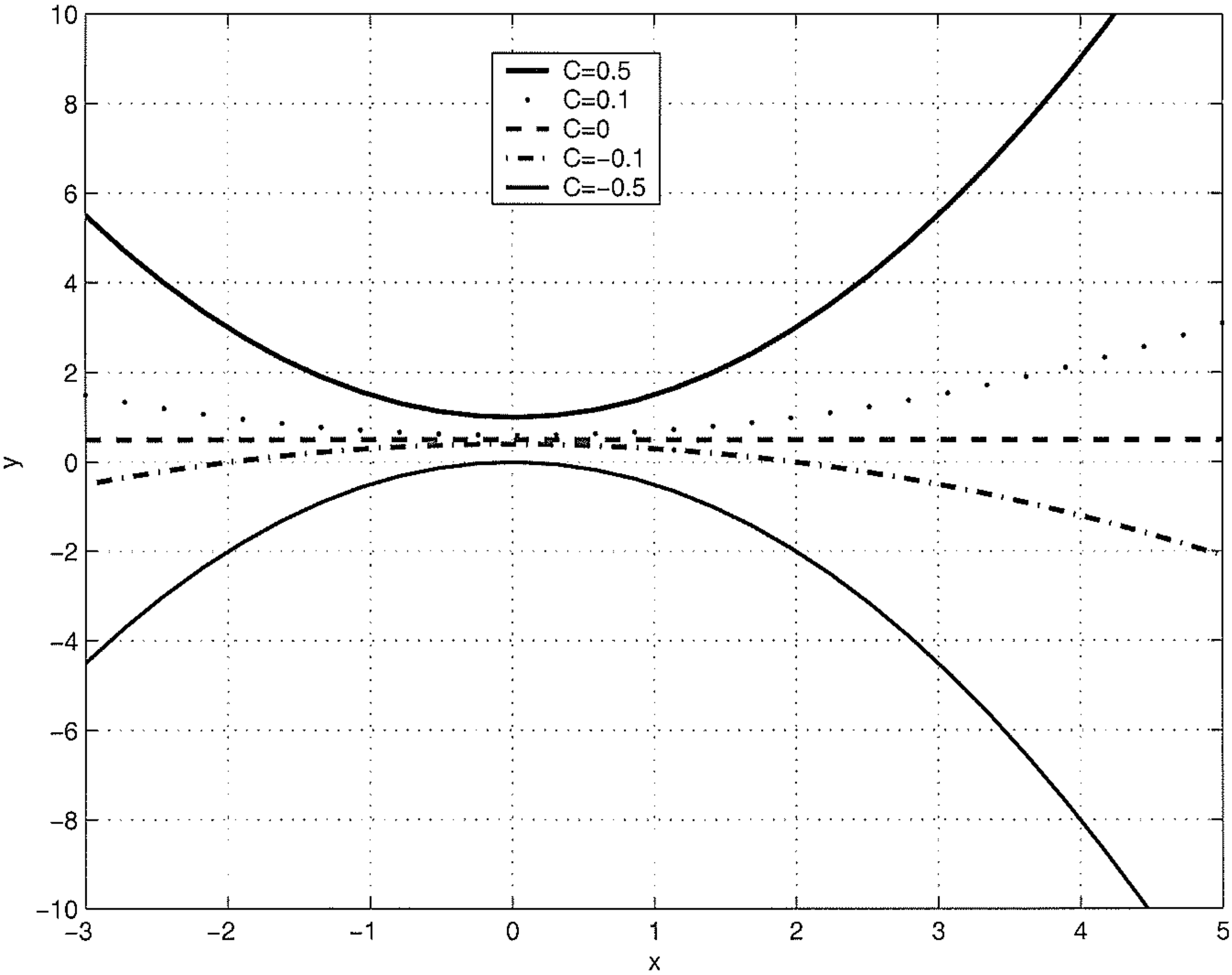




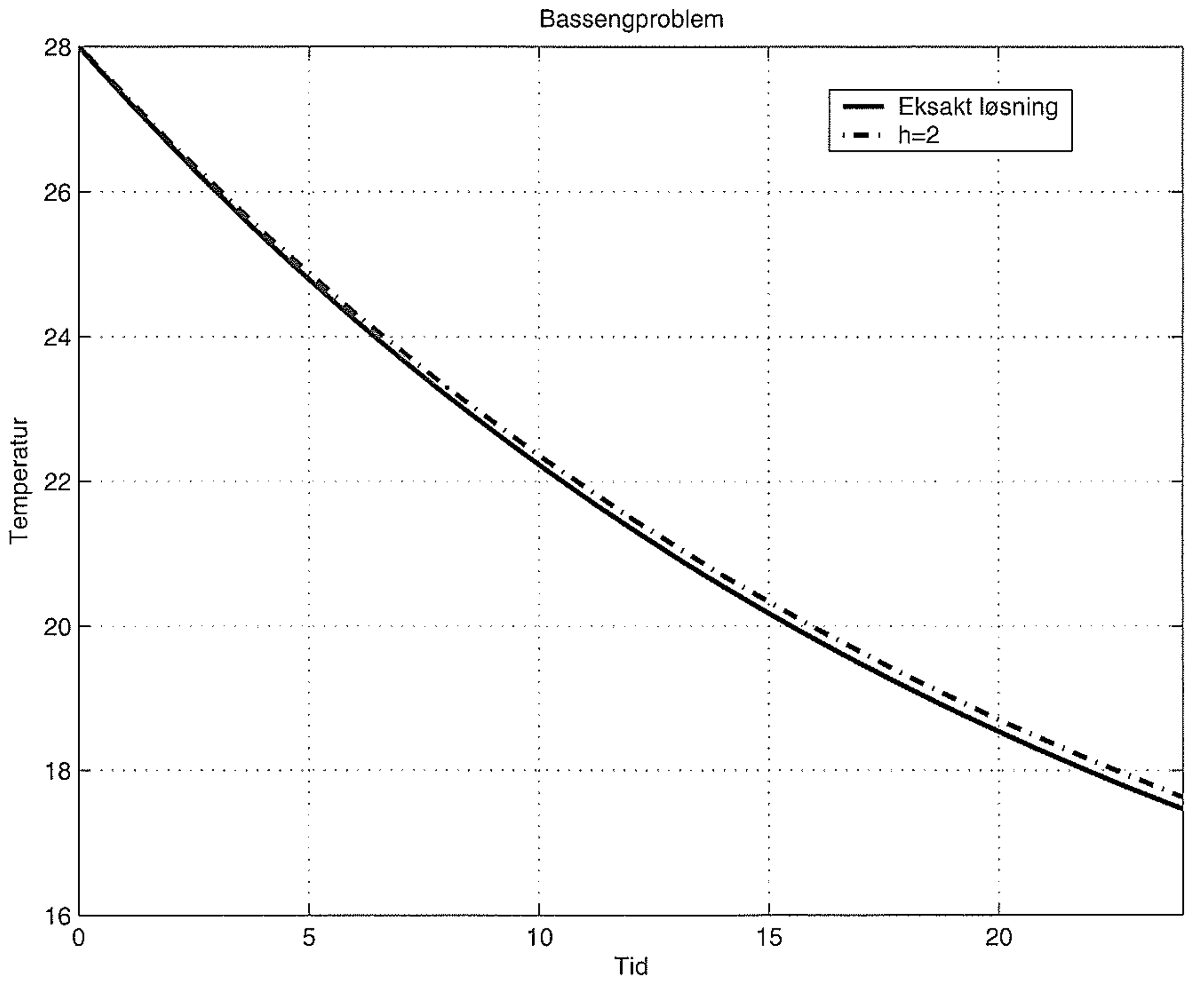
a=−1, b=1
y0=1
y0=0.5
y0=2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
y
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
t

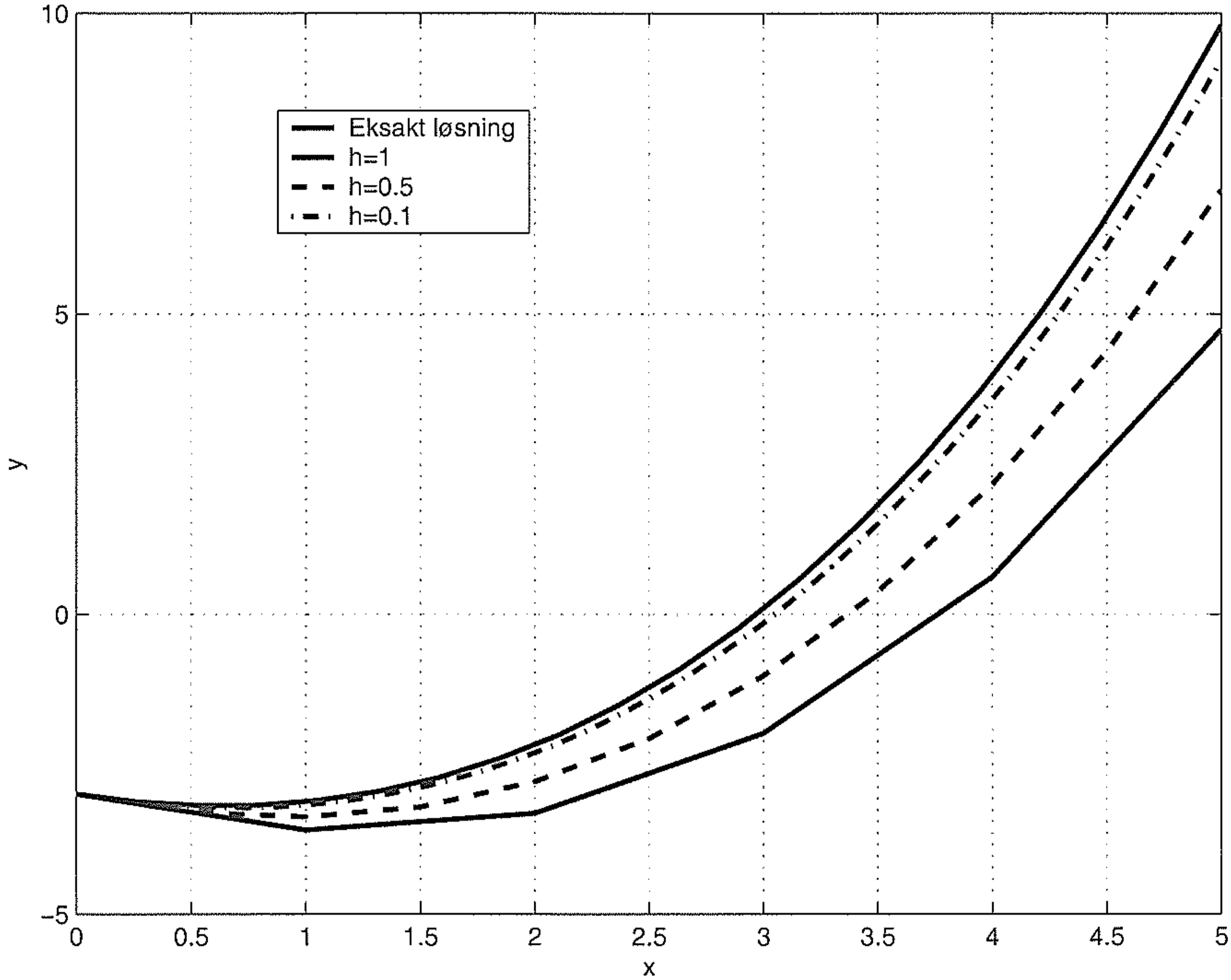


C=0.1
C=0
C=−0.1
C=0.5
y
x
10
8
6
4
2
0
−2
−4
−6
−8
−10
−3
−2
−1
0
1
2
3
4
5



C=0.5
C=0.1
C=0
C=−0.1
C=−0.5
10
8
6
4
2
0
−2
−4
−6
−8
−10
y
−3
−2
−1
0
1
2
3
4
5
x


Bassengproblem
Eksakt løsning
h=2
Temperatur
Tid
28
26
24
22
20
18
16
0
5
10
15
20



Eksakt løsning
h=1
h=0.5
h=0.1
10
5
0
−5
y
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
x