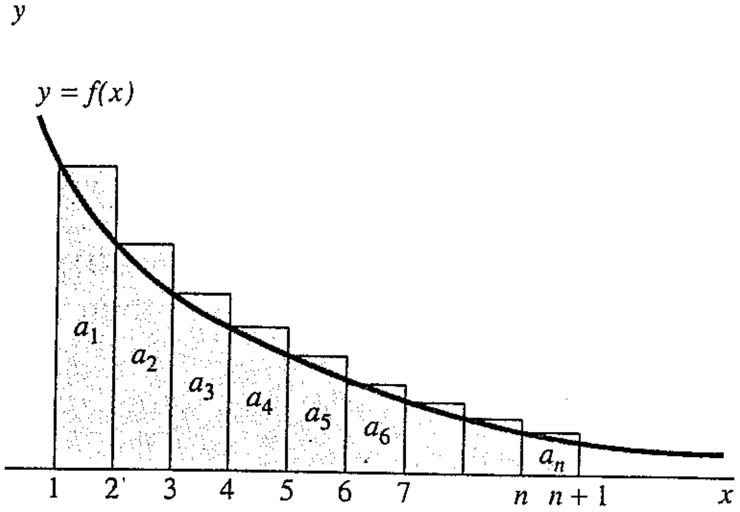
THEOREM 1 The Integral Test

Suppose that $\sum a_n$ is a positive-term series and that f is a positive-valued, decreasing, continuous function for $x \geqq 1$. If $f(n) = a_n$ for all integers $n \geqq 1$, then the series and the improper integral

$$\sum_{n=1}^{\infty} a_n \quad \text{and} \quad \int_1^{\infty} f(x)\,dx$$

either both converge or both diverge.



URE 11.5.1 Underestimating the partial sums
an integral.

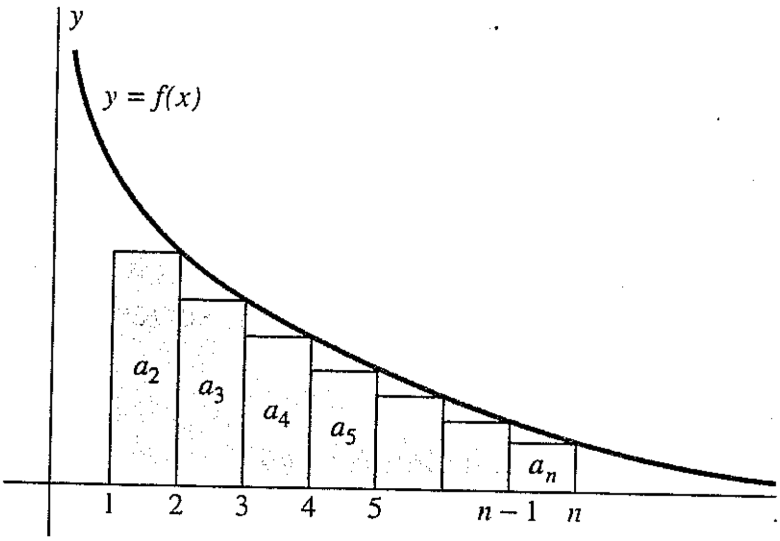


FIGURE 11.5.2 Overestimating the partial sums with an integral.

THEOREM 2 The Integral Test Remainder Estimate

Suppose that the infinite series and improper integral

$$\sum_{n=1}^{\infty} a_n \quad \text{and} \quad \int_1^{\infty} f(x)\,dx$$

satisfy the hypotheses of the integral test, and suppose in addition that both converge. Then

$$\int_{n+1}^{\infty} f(x)\,dx \leqq R_n \leqq \int_n^{\infty} f(x)\,dx,$$

where R_n is the remainder given in Eq. (5).

THEOREM 1 Comparison Test
Suppose that $\sum a_n$ and $\sum b_n$ are positive-term series. Then

1. $\sum a_n$ converges if $\sum b_n$ converges and $a_n \leqq b_n$ for all n;
2. $\sum a_n$ diverges if $\sum b_n$ diverges and $a_n \geqq b_n$ for all n.

THEOREM 2 Limit Comparison Test
Suppose that $\sum a_n$ and $\sum b_n$ are positive-term series. If the limit

$$L = \lim_{n\to\infty} \frac{a_n}{b_n}$$

exists and $0 < L < +\infty$, then either both series converge or both series diverge.

THEOREM 1 Alternating Series Test
If the alternating series in Eq. (1) satisfies the two conditions

1. $a_n \geqq a_{n+1} > 0$ for all n and
2. $\lim_{n\to\infty} a_n = 0$,

then the infinite series converges.

THEOREM 2 Alternating Series Remainder Estimate
Suppose that the series $\sum(-1)^{n+1}a_n$ satisfies the conditions of the alternating series test and therefore converges. Let S denote the sum of the series. Denote by $R_n = S - S_n$ the error made in replacing S with the nth partial sum S_n of the series. Then this **remainder** R_n has the same sign as the next term $(-1)^{n+2}a_{n+1}$ of the series, and

$$0 \leqq |R_n| < a_{n+1}.$$

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$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$$

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DEFINITION Absolute Convergence

The series $\sum a_n$ is said to **converge absolutely** (and is called **absolutely convergent** provided that the series

$$\sum |a_n| = |a_1| + |a_2| + |a_3| + \cdots + |a_n| + \cdots$$

converges.

THEOREM 3 Absolute Convergence Implies Convergence

If the series $\sum |a_n|$ converges, then so does the series $\sum a_n$.

THEOREM 4 The Ratio Test

Suppose that the limit

$$\rho = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right|$$

either exists or is infinite. Then the infinite series $\sum a_n$ of nonzero terms

1. Converges absolutely if $\rho < 1$;
2. Diverges if $\rho > 1$.

If $\rho = 1$, the ratio test is inconclusive.

THEOREM 5 The Root Test

Suppose that the limit

$$\rho = \lim_{n\to\infty} \sqrt[n]{|a_n|}$$

exists or is infinite. Then the infinite series $\sum a_n$

1. Converges absolutely if $\rho < 1$;
2. Diverges if $\rho > 1$.

If $\rho = 1$, the root test is inconclusive.