Oppgave 30
a) Observer at

$$\begin{aligned} A &= \int_0^h \sqrt{2py}\, dy \\ &= \frac{2}{3} h^{\frac{3}{2}} \sqrt{2p}. \end{aligned}$$

Nå er $r^2 = 2ph$, slik at

$$\begin{aligned} A &= \frac{2}{3} h h^{\frac{1}{2}} \sqrt{2p} \\ &= \frac{2}{3} h \sqrt{2ph} \\ &= \frac{2}{3} rh. \end{aligned}$$

Men

$$\begin{aligned} M_y &= \int_0^h \frac{1}{2}(2ph)\, dy \\ &= \frac{1}{2} ph^2, \end{aligned}$$

slik at

$$\begin{aligned} \bar{x} &= \frac{\frac{1}{2} ph^2}{\frac{2}{3} rh} \\ &= \frac{3ph}{4r}. \end{aligned}$$

Men $ph = \frac{1}{2} r^2$, s.a. $\bar{x} = \frac{3}{8} r$.
b)

$$\begin{aligned} V &= 2\pi \bar{x} A \\ &= 2\pi M_y \\ &= \pi ph^2. \end{aligned}$$

Men $ph = \frac{1}{2} r^2$, s.a.

$$V = \frac{1}{2} \pi r^2 h$$