FASIT, øvingsoppgaver

① a) $y = \sqrt{x^3} = (x^3)^{1/2} = x^{3/2}$

$\frac{dy}{dx} = \frac{3}{2} x^{1/2}$

b) $\frac{dy}{dx} = \frac{5}{2}(x^2+4x)^{3/2}(2x+4) = 5(x+2)(x^2+4x)^{3/2}$

c) $\frac{dy}{dx} = \frac{(4x^3+2x)(x^2+x+1)-(x^4+x^2)(2x+1)}{(x^2+x+1)}$ (forkort)

d) $\frac{dy}{dx} = 9\cdot(-1)x^{-2} = -9x^{-2}$

e) $y = \sqrt{\frac{1}{5x^6}} = \frac{1}{\sqrt{5x^6}} = \frac{1}{\sqrt{5}\cdot\sqrt{x^6}} = \frac{1}{\sqrt{5}(x^6)^{1/2}} = \frac{1}{\sqrt{5}x^3} = \frac{1}{\sqrt{5}}x^{-3}$

$\frac{dy}{dx} = -\frac{3}{\sqrt{5}}x^{-4}$

f) $\frac{dy}{dx} = \frac{1}{4}\left(12x + \frac{1}{2}x^{-1/2}\right)$

g) $\frac{dy}{dx} = \frac{1}{2\sqrt{1+\sin\sqrt{x}}}\cdot(\cos\sqrt{x})\cdot\frac{1}{2\sqrt{x}} = \frac{\cos\sqrt{x}}{4\sqrt{x}\sqrt{1+\sin\sqrt{x}}}$.

h) $\frac{dy}{dx} = \frac{6x^5+4x^3}{2\sqrt{x^6+x^4}} = \frac{2x^3(3x^2+2)}{2\sqrt{x^4(x^2+1)}} = \frac{x^3(3x^2+2)}{x^2\sqrt{x^2+1}}$

$= \frac{x(3x^2+2)}{\sqrt{x^2+1}}$.

② $f'(x) = 1 - \frac{1}{x^2} = 0 \Rightarrow x = \pm 1$ $\quad f(1) = 2$, $f(-1) = -2$

$f''(x) = \frac{2}{x^3}$, $f''(1) = 2 > 0 \Rightarrow (1,2)$ min. pkt.

$f''(-1) = -2 < 0 \Rightarrow (-1,-2)$ max. pkt.

④ Skjæringssetningen viser at det finns ett nullpunkt (minst ett). Hvis vi i tillegg kan vise at f er strengt voksende/avtagende får vi at det finns maks ett.

a) $-1,61$ b) $-0,51$

⑤ 3,141592654