

Oppgaver

1 Løs likningssystemet

$$\begin{array}{rcl} 3x_1 - x_2 - x_3 & = & 1 \\ -x_1 + 3x_2 & - & x_4 = 0 \\ -x_1 & + & 3x_3 - x_4 = 0 \\ -x_2 - x_3 + 3x_4 & = & 0 \end{array}$$

som best du kan.

Innssettingmetoden

$$\rightarrow x_2 = -x_3 + 3x_4$$

Matriser:
$$\left[\begin{array}{cccc|c} 3 & -1 & -1 & 0 & 1 \\ -1 & 3 & 0 & -1 & 0 \\ -1 & 0 & 3 & -1 & 0 \\ 0 & -1 & -1 & 3 & 0 \end{array} \right]$$

Gausseliminasjon:

- ① Bytte om på radene
- ② Gange en rad med et ikke-null tall
- ③ Gange en rad med et tall $\neq 0$ og legg til en annen rad
(e.g. erstatt rad 1 med rad 1 + 3·rad 2, rad 2 stå uendret)

Mål: trappeform

$$\left[\begin{array}{cccc|c} \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot & \cdot \end{array} \right]$$

hvis mulig:
$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \cdot \\ 0 & 1 & 0 & 0 & \cdot \\ 0 & 0 & 1 & 0 & \cdot \\ 0 & 0 & 0 & 1 & \cdot \end{array} \right]$$

2 Løs likningssystemet

$$\begin{array}{rcl} 3x_1 - x_2 - x_3 & = & 1 \\ -x_1 + 3x_2 & - & x_4 = 0 \\ -x_1 & + & 3x_3 - x_4 = 0 \\ -x_2 - x_3 + 3x_4 & = & 0 \end{array}$$

med gausseliminasjon.

$$\begin{aligned} & \left[\begin{array}{cccc|c} 3 & -1 & -1 & 0 & 1 \\ -1 & 3 & 0 & -1 & 0 \\ -1 & 0 & 3 & -1 & 0 \\ 0 & -1 & -1 & 3 & 0 \end{array} \right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{cccc|c} -1 & 3 & 0 & -1 & 0 \\ 3 & -1 & -1 & 0 & 1 \\ -1 & 0 & 3 & -1 & 0 \\ 0 & -1 & -1 & 3 & 0 \end{array} \right] \xrightarrow{R_1 \cdot (-1)} \left[\begin{array}{cccc|c} 1 & -3 & 0 & 1 & 0 \\ 3 & -1 & -1 & 0 & 1 \\ -1 & 0 & 3 & -1 & 0 \\ 0 & -1 & -1 & 3 & 0 \end{array} \right] \xrightarrow{\begin{array}{l} R_2 - 3R_1 \\ R_3 + R_1 \end{array}} \\ & \sim \left[\begin{array}{cccc|c} 1 & -3 & 0 & 1 & 0 \\ 0 & 8 & -1 & -3 & 1 \\ 0 & -3 & 3 & 0 & 0 \\ 0 & -1 & -1 & 3 & 0 \end{array} \right] \xrightarrow{R_2 \leftrightarrow R_4} \left[\begin{array}{cccc|c} 1 & -3 & 0 & 1 & 0 \\ 0 & -1 & -1 & 3 & 0 \\ 0 & -3 & 3 & 0 & 0 \\ 0 & 8 & -1 & -3 & 1 \end{array} \right] \xrightarrow{R_2 \cdot (-1)} \left[\begin{array}{cccc|c} 1 & -3 & 0 & 1 & 0 \\ 0 & 1 & 1 & -3 & 0 \\ 0 & -3 & 3 & 0 & 0 \\ 0 & 8 & -1 & -3 & 1 \end{array} \right] \xrightarrow{\begin{array}{l} R_1 + 3R_2 \\ R_3 + 3R_2 \\ R_4 - 8R_2 \end{array}} \end{aligned}$$

$$\begin{aligned}
 &\sim \left[\begin{array}{cccc|c} 1 & 0 & 3 & -8 & 0 \\ 0 & 1 & 1 & -3 & 0 \\ 0 & 0 & 6 & -9 & 0 \\ 0 & 0 & -9 & 21 & 1 \end{array} \right] R_3 \cdot \frac{1}{6} \sim \left[\begin{array}{cccc|c} 1 & 0 & 3 & -8 & 0 \\ 0 & 1 & 1 & -3 & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 0 \\ 0 & 0 & -9 & 21 & 1 \end{array} \right] \begin{array}{l} R_1 - 3R_3 \\ R_2 - R_3 \\ \sim \\ R_4 + 9R_3 \end{array} \left[\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{7}{2} & 0 \\ 0 & 1 & 0 & -\frac{3}{2} & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & \frac{15}{2} & 1 \end{array} \right] R_4 \cdot \frac{2}{15} \\
 &\sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & -\frac{7}{2} & 0 \\ 0 & 1 & 0 & -\frac{3}{2} & 0 \\ 0 & 0 & 1 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 & 1 & \frac{2}{15} \end{array} \right] \begin{array}{l} R_1 + \frac{7}{2}R_4 \\ R_2 + \frac{3}{2}R_4 \\ R_3 + \frac{3}{2}R_4 \end{array} \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & \frac{7}{15} \\ 0 & 1 & 0 & 0 & \frac{1}{5} \\ 0 & 0 & 1 & 0 & \frac{1}{5} \\ 0 & 0 & 0 & 1 & \frac{2}{15} \end{array} \right] \Rightarrow \begin{array}{l} x_1 = \frac{7}{15} \\ x_2 = \frac{1}{5} \\ x_3 = \frac{1}{5} \\ x_4 = \frac{2}{15} \end{array}
 \end{aligned}$$

3 Løs likningssystemet

$$\begin{array}{rcl}
 3x_1 - x_2 - x_3 & = & 0 \\
 -x_1 + 3x_2 & - & x_4 = 0 \\
 -x_1 & + & 3x_3 - x_4 = 0 \\
 -x_2 - x_3 + 3x_4 & = & 0
 \end{array}$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right] \Rightarrow x_1 = x_2 = x_3 = x_4 = 0$$

4 Amund har høns og kyr på gården sin. Vrang av vane vil han ikke si hvor mange dyr han har, men opplyser heller at de har 382 bein og 141 hoder. Hvor mange høns og hvor mange kyr har Amund?

x = antall høns

y = antall kyr

$$\begin{array}{rcl}
 x + y & = & 141 \\
 2x + 4y & = & 382
 \end{array}$$

$$\leadsto \left[\begin{array}{cc|c} 1 & 1 & 141 \\ 2 & 4 & 382 \end{array} \right]$$

$$\Rightarrow x = 91, y = 50$$

5 Løs likningssystemet

$$\begin{array}{rcl}
 3x_1 - x_2 - x_3 & = & 0 \\
 -x_1 + 3x_2 & - & x_4 = 0 \\
 -x_1 & + & 3x_3 - x_4 = 0 \\
 x_1 + 2x_2 + 2x_3 + x_4 & = & 0
 \end{array}$$

6 Løs likningssystemet

$$\begin{array}{rcl}
 3x_1 - x_2 - x_3 & = & 1 \\
 -x_1 + 3x_2 & - & x_4 = 1 \\
 -x_1 & + & 3x_3 - x_4 = 1 \\
 x_1 + 2x_2 + 2x_3 + x_4 & = & 3
 \end{array}$$

7 Løs likningssystemet

$$\begin{array}{rcl}
 3x_1 - x_2 - x_3 & = & 1 \\
 -x_1 + 3x_2 & - & x_4 = 1 \\
 -x_1 & + & 3x_3 - x_4 = 1 \\
 x_1 + 2x_2 + 2x_3 + x_4 & = & 1
 \end{array}$$