FASIT, Lørdagsverksted 12. mars

① a) $\frac{(x-1)^2}{2^2} + \frac{(y-2)^2}{3^2} = 1$ Ellipse.

b) $x - 25 = -(y+5)^2$ Parabel.

c) $y + 19 = 2(x-3)^2$ Parabel.

② a) $\cos t = \frac{x-1}{2}$ $\sin t = \frac{y-2}{3}$

$x = 1 + 2\cos t$ $y = 2 + 3\sin t$; $0 \le t \le 2\pi$.

b) $-x^2 + \frac{(y-5)^2}{2^2} = 1$

$x = \sinh t$ $y = 5 + 2\cosh t$

c) $y = 1 + t$, $x = 3 + 2t^2$ $-\infty < t < \infty$.

③ a) $(\sqrt{2}, \pi/4)$

b) $(2, \pi/3)$

c) $(5, 0.927)$

⑤ b) $L = \sqrt{2}\,(e^{2\pi} - 1)$.

⑥ a) $f_{xx} = 42x^5y^3$, $f_{xy} = f_{yx} = 21x^6y^2$, $f_{yy} = 6x^7y$

b) $f_{xx} = \frac{2x^3 - 6xy^2}{(x^2+y^2)^3}$, $f_{xy} = f_{yx} = \frac{6x^2y - 2y^3}{(x^2+y^2)^3}$, $f_{yy} = \frac{6xy^2 - 2x^3}{(x^2+y^2)^3}$

c) $f_{xx} = \frac{1+y^2}{(1+x^2+y^2)^{3/2}}$, $f_{xy} = f_{yx} = \frac{-xy}{(1+x^2+y^2)^{3/2}}$, $f_{yy} = \frac{1+x^2}{(1+x^2+y^2)^{3/2}}$

⑦ Sett prøve på ligningen.

⑧ a) $\frac{1}{(x^2+y^2)^2}[y^2 - x^2, -2xy]$

b) $e^{x^2+yz}[2x, z, y]$

⑨ a) $z = 2(e - \frac{1}{e})(x-1) + \frac{1}{2}(e + \frac{1}{e})y$

b) $z = 4x + y - 3$

c) $z = \frac{2}{5}x + \frac{2}{5}y + 1$

⑩ a) $\frac{4e^{2t}}{\sqrt{4e^{4t}+1}}$

b) $\frac{2t + 4t^3 + 1}{2\sqrt{t^2 + t^4 + t}}$

(11) a) $(1,1)$ sadelpunkt

b) $(0,2)$ lokalt min., $(\pm\frac{1}{2}, \frac{3}{2})$ sadelpunkter

c) $(0,0)$ sadelpunkt

Eksamensoppgaver

(1) $(1,1), (1,-1), (-1,1), (-1,-1)$

Maksimalverdi : $f(1,1) = f(-1,-1) = e^{-1}$

Minimalverdi : $f(1,-1) = f(-1,1) = -e^{-1}$.

(2) $S(y,\theta) = \frac{1}{y} + \frac{2-\cos\theta}{\sin\theta}\, y$

$S_{min} = 2 \cdot 3^{1/4}$ for $x = \frac{2}{3} \cdot 3^{1/4}$, $y = 3^{-1/4}$, $\theta = \pi/3$.