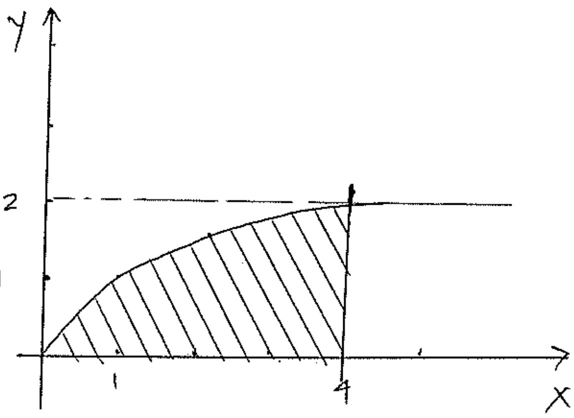
EKSAMENSOPPGAVER

(1) $$\int_0^2 \int_{y^2}^4 \cos\sqrt{x^3}\,dx\,dy$$ (Bytt integrasjonsrekkefølge)



Ved hjelp av figuren endrer vi grensene til

$$\int_0^4 \int_0^{\sqrt{x}} \cos\sqrt{x^3}\,dy\,dx$$

som kan skrives som $$\underline{\int_0^4 \int_0^{x^{1/2}} \cos x^{3/2}\,dy\,dx}$$

Da får vi

$$\int_0^4 \Big[y \cos x^{3/2} \Big]_0^{x^{1/2}} dx = \int_0^4 x^{1/2} \cos x^{3/2}\,dx$$

Substitusjon $u = x^{3/2} \Rightarrow du = \frac{3}{2} x^{1/2}\,dx$

som gir oss

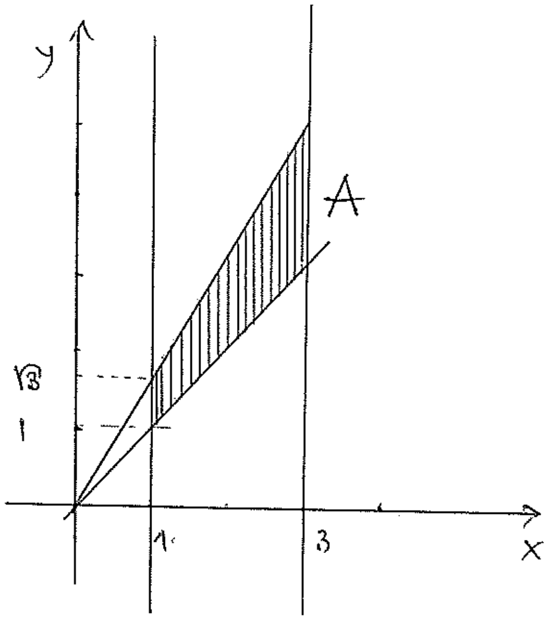
$$\int_0^8 \frac{2}{3} x^{1/2} \cos u \frac{du}{x^{1/2}} = \frac{2}{3} \int_0^8 \cos u\,du$$

$$\Rightarrow \frac{2}{3} \Big[\sin u \Big]_0^8 = \underline{\underline{\frac{2}{3} \sin 8}}$$

(2) A er området begrensa av linjene

$y=x$, $y=\sqrt{3}\,x$, $x=1$, $x=3$.

med tetthet $\rho(x,y)=(x^2+y^2)^{-3/2}$



Massen:

$$M=\iint_A \rho(x,y)\,dA$$

$$M=\iint_A (x^2+y^2)^{-3/2}\,dA$$

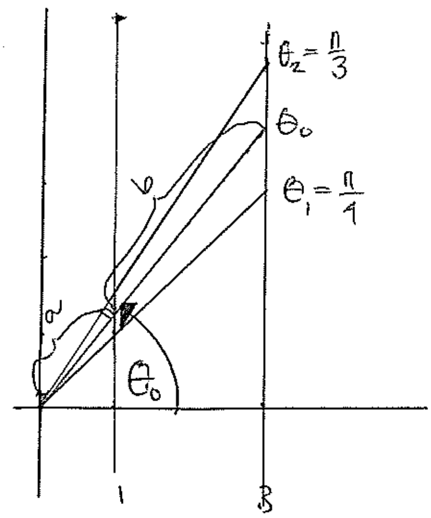
Vi skal innføre polarkoordinater, r og θ.

$x=r\cos\theta$
$y=r\sin\theta$

$$M=\iint_A (r^2)^{-3/2}\,r\,dr\,d\theta=\iint_A r^{-2}\,dr\,d\theta$$

Vi må ha nye grenser.

$\frac{\pi}{4}\le\theta\le\frac{\pi}{3}$ $\qquad a\le r\le a+b$



Uttrykker r ved θ vha. figuren og trigonometri.

$$\frac{1}{a}=\cos\theta_0\,,\quad \frac{3}{a+b}=\cos\theta_0$$

$$\Rightarrow$$

$$\frac{1}{\cos\theta}\le r\le\frac{3}{\cos\theta}$$

Integralet i polarkoordinater blir da

$$\int_{\pi/4}^{\pi/3} \int_{\frac{1}{\cos\theta}}^{\frac{3}{\cos\theta}} r^{-2} \, dr \, d\theta$$

$$= \int_{\pi/4}^{\pi/3} \left[-\frac{1}{r} \right]_{r=\frac{1}{\cos\theta}}^{r=\frac{3}{\cos\theta}} d\theta = \int_{\pi/4}^{\pi/3} \left(-\frac{\cos\theta}{3} + \cos\theta \right) d\theta$$

$$= \frac{2}{3} \int_{\pi/4}^{\pi/3} \cos\theta \, d\theta = \frac{2}{3} \Big[\sin\theta \Big]_{\pi/4}^{\pi/3} = \frac{2}{3} \left[\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \right]$$

$$= \underline{\underline{\frac{\cancel{4}}{3} \left(\sqrt{3} - \sqrt{2} \right)}}$$