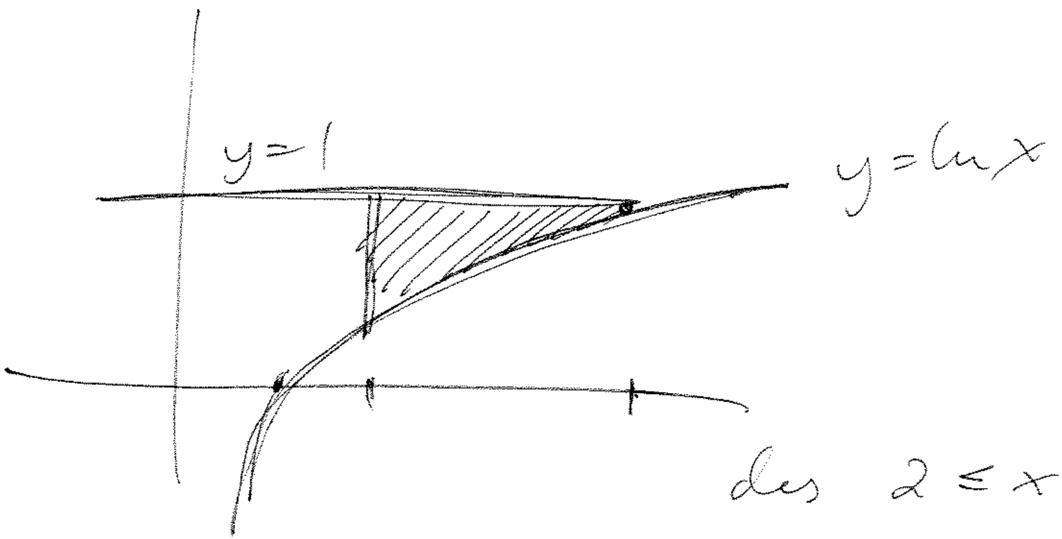
Aug 98 oppg 3

$$I = \int_2^e \int_{\ln x}^1 \frac{2xy}{e^y+2}\,dy\,dx$$

$$\ln x \le y \le 1$$
$$2 \le x \le e$$



dvs $2 \le x \le e^y$
$\ln 2 \le y \le 1$

Så $$I = \int_{\ln 2}^{1} \int_2^{e^y} \frac{2xy}{e^y+2}\,dx\,dy$$

$$= \int_{\ln 2}^{1} \left[\frac{x^2 y}{e^y+2} \right]_{x=2}^{x=e^y} dy$$

$$= \int_{\ln 2}^{1} \frac{(e^{2y}-4)y}{e^{y}+2}\,dy = \int_{\ln 2}^{1} \frac{(e^{y}+2)(e^{y}-2)y\,dy}{e^{y}+2}$$

$$= \int_{\ln 2}^{1} (e^{y}-2)y\,dy = \left[ye^{y}-e^{y}-y^{2}\right]_{\ln 2}^{1}$$

$$= -1 - 2\ln 2 + 2 + (\ln 2)^2$$

$$= \underline{(1-\ln 2)^2}$$

Aug 98 oppg 5

$$S_1 : \quad z = 9 - x^2 - (y-1)^2$$

$$S_2 : \quad z = 2y + 4$$

a) På skjæringskurven har vi

$$2y + 4 = 9 - x^2 - y^2 + 2y - 1 \quad \text{dvs}$$

$$x^2 + y^2 = 4$$

Så projeksjonen i xy-planet blir

sirkelen $x^2 + y^2 = 4$.

T blir da beskrevet av

$$2y + 4 \le z \le 9 - x^2 - (y-1)^2$$

$$x^2 + y^2 \le 4$$

Så

$$\text{vol}(T) = \iiint_T 1 \, dV = \iint_{x^2+y^2 \le 4} \int_{2y+4}^{9-x^2-(y-1)^2} dz \, dx dy$$

$$= \iint_{x^2+y^2 \le 4} (9 - x^2 - (y-1)^2 - 2y - 4) \, dx dy = \iint_{x^2+y^2 \le 4} (4 - x^2 - y^2) \, dx dy$$

Polar koord gir:

$$= \int_0^{2\pi} \int_0^2 (4 - r^2) r \, dr d\theta = 2\pi \left[2r^2 - \tfrac{1}{4} r^4 \right]_0^2$$

$$= \underline{8\pi}$$

c) Stokes' teorem gir at

$$\int_C \vec{F}\cdot\vec{T}\,ds = \iint_{S_2} \mathrm{curl}(\vec{F})\cdot\vec{n}\,dS$$

$$\mathrm{curl}(\vec{F}) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x-yz & 0 & 0 \end{vmatrix} = (0,-y,z)$$

Som i b) beskrives S_2 ved

$$\vec{r}(x,y) = (x,\ y,\ 2y+4)$$

$$\vec{r_x}(x,y) = (1,0,0) \qquad \vec{r_y}(x,y) = (0,1,2)$$

$$\mathrm{curl}(\vec{F})\cdot(\vec{r_x}\times\vec{r_y}) = \begin{vmatrix} 0 & -y & 2y+4 \\ 1 & 0 & 0 \\ 0 & 1 & 2 \end{vmatrix}$$

$= 4y+4$ OBS: $\vec{r_x}\times\vec{r_y}$ peker riktig vei !

Så $$\iint_{S_2} \mathrm{curl}(\vec{F})\cdot\vec{n}\,dS = \iint_{x^2+y^2\le 4} \mathrm{curl}(\vec{F})\cdot(\vec{r_x}\times\vec{r_y})\,dx\,dy$$

$$= \iint_{x^2+y^2\le 4} (4y+4)\,dx\,dy = 4\cdot \text{areal}(x^2+y^2\le 4)$$

$$= \underline{\underline{16\pi}}$$

b) I følge divergensteoremet er

$$\iint_S \vec{F} \cdot \vec{n}\, dS = \iiint_T \operatorname{div} \vec{F}\, dV$$

$$= \iiint_T 2\, dV = 2 \operatorname{vol}(T) = \underline{16\pi}$$

Over $S_2 = A$ er $\vec{n} = \frac{1}{\sqrt{5}}(0, 2, -1)$

dvs $\vec{F} \cdot \vec{n} = 0$ og

$$\iint_{S_2} \vec{F} \cdot \vec{n}\, dS = \underline{0}$$

$$\iint_S \vec{F} \cdot \vec{n}\, dS = \iint_{S_1} \vec{F} \cdot \vec{n}\, dS + \iint_{S_2} \vec{F} \cdot \vec{n}\, dS$$

dvs $\iint_{S_1} \vec{F} \cdot \vec{n}\, dS = \underline{16\pi}$