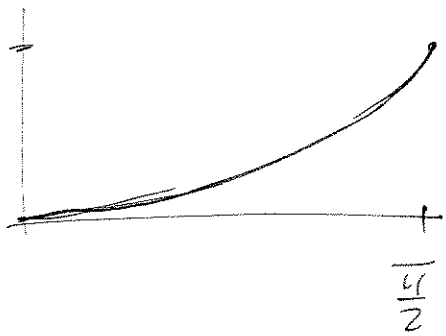
Aug 99. oppg 1

$$\vec{r}(t) = (t + \sin t \cos t,\ \sin^2 t)$$

$$\vec{r}\,'(t) = (1 + \cos^2 t - \sin^2 t,\ 2 \sin t \cos t)$$

$$= (1 + \cos 2t,\ \sin 2t)$$



$$|\vec{r}\,'(t)| = \sqrt{1 + 2\cos 2t + \cos^2 2t + \sin^2 2t}$$

$$= \sqrt{2(1 + \cos 2t)} = 2|\cos t|$$

Så $L = \int_0^{\frac{\pi}{2}} 2|\cos t|\,dt = \left[2 \sin t\right]_0^{\frac{\pi}{2}} = \underline{2}$

b) $\vec{r}\left(\frac{\pi}{6}\right) = \left(\frac{\pi}{6} + \frac{\sqrt{3}}{4},\ \frac{1}{4}\right)$

$$\vec{\nabla} T(x, y) = (2x,\ 2y)$$

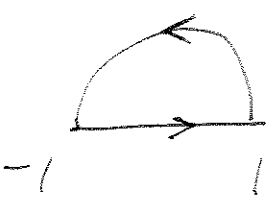
$$\vec{\nabla} T\left(\frac{\pi}{6} + \frac{\sqrt{3}}{4},\ \frac{1}{4}\right) = \left(\frac{\pi}{3} + \frac{\sqrt{3}}{2},\ \frac{1}{2}\right)$$

$$\vec{r}\,'\left(\frac{\pi}{6}\right) = \left(1 + \frac{1}{2},\ \frac{\sqrt{3}}{2}\right) = \frac{1}{2}(3, \sqrt{3})$$

Temp. endring pr. lengdeenhet blir

$$\vec{\nabla} T \cdot \frac{\vec{r}\,'}{|\vec{r}\,'|} = \left(\frac{\pi}{3} + \frac{\sqrt{3}}{2},\ \frac{1}{2}\right) \cdot \left(\frac{\sqrt{3}}{2},\ \frac{1}{2}\right) = \underline{\frac{\pi\sqrt{3}}{6} + 1}$$

Oppg 3



$$\Phi = \oint_C \vec{F}\cdot\vec{T}\,ds = \iint_R \text{curl}(\vec{F})\cdot\vec{k}\,dxdy$$

$$= \iint_R \frac{\partial}{\partial x}\left(\frac{x^2}{2}\right) - \frac{\partial}{\partial y}\left(-\frac{y^2}{2}\right)\,dxdy$$

$$= \iint_R (x+y)\,dxdy = \iint_R y\,dxdy$$

Polarkoord: $x = r\cos\theta \quad y = r\sin\theta$

$0 \le r \le 0 \quad 0 \le \theta \le \pi$ så

$$\Phi = \int_0^{\pi}\int_0^{1} r\sin\theta\; r\,dr\,d\theta = \int_0^{\pi}\left[\frac{1}{3}r^3\sin\theta\right]_{r=0}^{r=1} d\theta$$

$$= \int_0^{\pi}\frac{1}{3}\sin\theta\,d\theta = \left[-\frac{1}{3}\cos\theta\right]_0^{\pi} = \underline{\frac{2}{3}}$$

b) Over L er $y=0$ $\quad \vec{F}(x,0) = \left(0, \frac{x^2}{2}\right)$

$\vec{n} = (0,-1)$ så $\vec{F}\cdot\vec{n} = -\frac{x^2}{2}$

dvs fluksen over L er $\int_L \vec{F}\cdot\vec{n}\,ds = \int_{-1}^{1} -\frac{x^2}{2}\,dx = \underline{-\frac{1}{3}}$

Siden $\int_C \vec{F}\cdot\vec{n}\,ds = \int_{C_1}\vec{F}\cdot\vec{n}\,ds + \int_L \vec{F}\cdot\vec{n}\,ds$

$= \iint_R \text{div}\vec{F}\,dxdy = 0$ så $\int_{C_1}\vec{F}\cdot\vec{n}\,ds = \frac{1}{3}$

[redacted]

5a

Volum: $V = \iint_R \left(\sqrt{2y} - \sqrt{x^2+y^2}\right) dx\,dy$

Polarkoordinater: $x = r\cos\theta \quad y = r\sin\theta$

$R: x^2+y^2 \le 2y \iff r^2 \le 2r\sin\theta \iff 0 \le r \le 2\sin\theta$

$$V = \int_0^{\pi}\int_0^{2\sin\theta} \left(\sqrt{2r\sin\theta} - r\right) r\,dr\,d\theta$$

$$= \int_0^{\pi} \left[\sqrt{2\sin\theta}\ \frac{2}{5} r^{\frac{5}{2}} - \frac{1}{3} r^3\right]_{r=0}^{r=2\sin\theta} d\theta$$

$$= \int_0^{\pi} \left(\frac{2}{5} - \frac{1}{3}\right)(2\sin\theta)^3\,d\theta = \int_0^{\pi} \frac{8}{15}(1-\cos^2\theta)\sin\theta$$

$$= \frac{8}{15}\left[-\cos\theta + \frac{1}{3}\cos^3\theta\right]_0^{\pi} = \frac{32}{45}$$

OBS Skjæringskurven: $z^2 = x^2+y^2 = 2y$

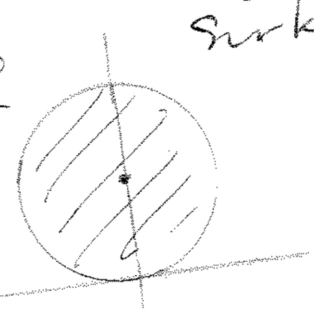
dvs $x^2+y^2 = 2y \iff x^2 + (y-1)^2 = 1$

sirkel med radius 1, sentrum $(0,1)$

I polarkoord: $r^2 = 2r\sin\theta$

dvs $r = 2\sin\theta$

R



5b)

$$\vec{r}(r,\theta) = (r\cos\theta, r\sin\theta, r)$$

$$\frac{\partial \vec{r}}{\partial r} = (\cos\theta, \sin\theta, 1)$$

$$\frac{\partial r}{\partial \theta} = (-r\sin\theta, r\cos\theta, 0)$$

$$\frac{\partial r}{\partial r} \times \frac{\partial r}{\partial \theta} = (-r\cos\theta, -r\sin\theta, r)$$

$$dS = \left|\frac{\partial \vec{r}}{\partial r} \times \frac{\partial \vec{r}}{\partial \theta}\right| = \sqrt{r^2\cos^2\theta + r^2\sin^2\theta + r^2} = \sqrt{2}\, r$$

Area: $$\iint_{r\leq 2\sin\theta} dS = \int_0^{\pi}\int_0^{2\sin\theta} \sqrt{2}\, r\, dr\, d\theta$$

$$= \int_0^{\pi} \left[\frac{\sqrt{2}}{2} r^2\right]_0^{2\sin\theta} d\theta = \int_0^{\pi} 2\sqrt{2}\sin^2\theta\, d\theta$$

$$= 2\sqrt{2}\int_0^{\pi} \frac{1-\cos 2\theta}{2}\, d\theta = \sqrt{2}\,\pi$$

jc **Alt 1** C kan parametrisert som

$r = 2\sin\theta \quad z = r$ dvs

$$\vec{r}(\theta) = \left(2\cos\theta\sin\theta,\ 2\sin^2\theta,\ 2\sin\theta\right) \quad 0 \leq \theta \leq \pi.$$

Se på

$$f(\theta) = \left|\vec{r}(\theta) - (0,0,2)\right|^2$$

$$= 4\cos^2\theta\sin^2\theta + 4\sin^4\theta + (2\sin\theta - 2)^2$$

$$= 8\sin^2\theta - 8\sin\theta + 4$$

$$f'(\theta) = 16\cos\theta\sin\theta - 8\cos\theta$$

$$= 8\cos\theta(-1 + 2\sin\theta)$$

$f'(\theta) = 0 \iff \cos\theta = 0$ eller $\sin\theta = \frac{1}{2}$

i) $\cos\theta = 0 \iff \theta = \frac{\pi}{2} \quad f\left(\frac{\pi}{2}\right) = 4$

ii) $\sin\theta = \frac{1}{2} \iff \theta = \frac{\pi}{6}, \frac{5\pi}{6} \quad f(\theta) = 8\cdot\frac{1}{4} - 8\frac{1}{2} + 4 =$

dvs $\left(\pm\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$ ligger nærmes

~~c)~~ ~~A02~~

Finn min av

$f(x,y,z) = x^2+y^2+(z-2)^2$

(1) når $g(x,y,z) = x^2+y^2-z^2 = 0$

(2) og $h(x,y,z) = z^2-2y = 0$

Lagrange gir:

$\vec{\nabla} f = \lambda \vec{\nabla} g + \mu \vec{\nabla} h$ dvs

$$\left(2x, 2y, 2(z-2)\right) = \lambda\left(2x, 2y, -2z\right) + \mu\left(0, -2, 2z\right)$$

dvs

(1) $2x = 2\lambda x$

(2) $2y = 2\lambda y - 2\mu$

(3) $2z-4 = -2\lambda z + 2\mu z$

(4) $x^2+y^2-z^2 = 0$

(5) $z^2-2y = 0$

Løs dette, og vi får $\lambda = 1$ $\mu = 0$

$x = \pm\frac{\sqrt{3}}{2}$ $y = \frac{1}{2}$ $z = 1$, så

begge punktene $\left(\pm\frac{\sqrt{3}}{2}, \frac{1}{2}, 1\right)$ ligger nærmest

5d)

$$\mathrm{curl}(\vec{F}) = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & e^{z^2} & xz \end{vmatrix}$$

$$= \left(-2ze^{z^2}, -z, 1\right)$$

Alt 1)

Stokes' teorem gir

$$I = \oint_C \vec{F} \cdot \vec{T}\, ds = \iint_S \mathrm{curl}(\vec{F}) \cdot \vec{n}\, dS$$

der f.eks S er flaten gitt ved

$z^2 = 2y$, dvs

$\vec{r}(x,y) = (x, y, \sqrt{2y})$ $\quad (x,y) \in R$

SJEKK at $\vec{n}$ peker riktig vei!

R = sirkelen fra 5a). Så

$$I = \iint_R \mathrm{curl}(\vec{F}) \cdot (\vec{r}_x \times \vec{r}_y)\, dx\,dy$$

$$= \iint_R \begin{vmatrix} -2\sqrt{2}\sqrt{y}\, e^{2y} & -\sqrt{2y} & 1 \\ 1 & 0 & 0 \\ 0 & 1 & \frac{1}{\sqrt{2y}} \end{vmatrix} dx\,dy$$

$$= 2\, \mathrm{areal}(R) = \underline{2\pi}$$

5d

Alt 2 Fra 5c har vi at
C kan parametriseres som

$$\vec{r}(\theta) = (2\cos\theta\sin\theta,\ 2\sin^2\theta,\ 2\sin\theta)$$

$$\vec{r}'(\theta) = (2\cos 2\theta,\ 2\sin 2\theta,\ 2\cos\theta) \qquad 0 \le \theta \le \pi$$

der

$$I = \int_C \vec{F}\cdot\vec{T}\,ds = \int_0^{\pi} \vec{F}(\vec{r}(\theta))\cdot\vec{r}'(\theta)\,d\theta$$

$$= \int (-2\sin^2\theta,\ e^{4\sin^2\theta},\ 4\cos\theta\sin^2\theta)\cdot 2(\cos 2\theta, \sin 2\theta, \cos\theta)\,d\theta$$

$$= \int_0^{\pi} \left(4\sin^2\theta + 4e^{4\sin^2\theta}\sin\theta\cos\theta\right)d\theta$$

$$I_1 = \int_0^{\pi} 4\sin^2\theta\,d\theta = \int_0^{\pi} 2(1-\cos 2\theta)\,d\theta = \underline{2\pi}$$

$$I_2 = \int_0^{\pi} 4e^{4\sin^2\theta}\sin\theta\cos\theta\,d\theta = \left[\tfrac{1}{2}e^{4\sin^2\theta}\right]_0^{\pi} = \underline{0}$$

Så $I = I_1 + I_2 = \underline{2\pi}$