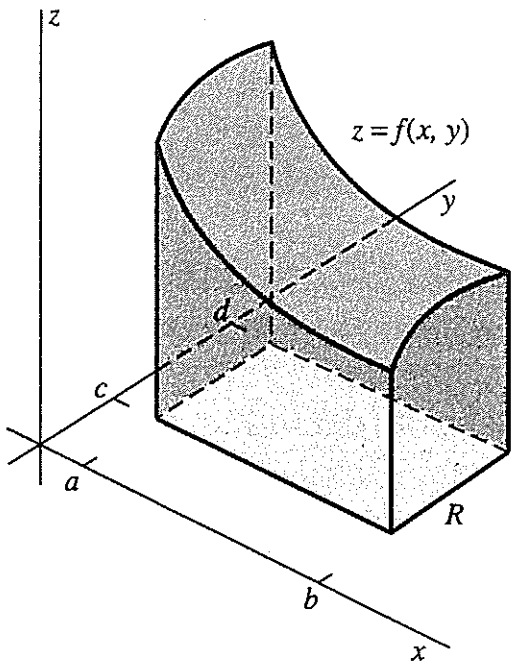
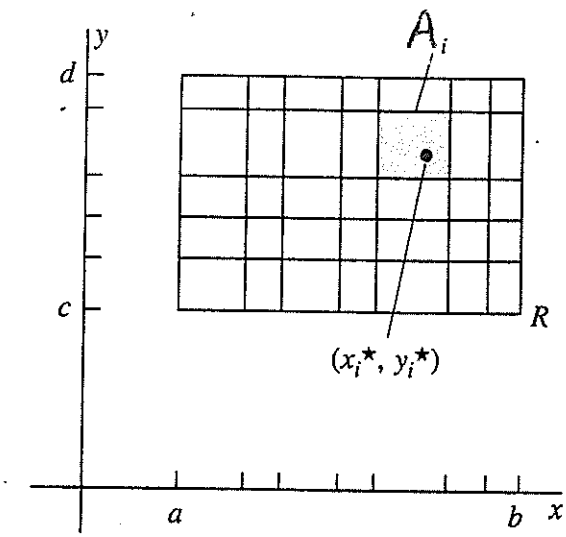
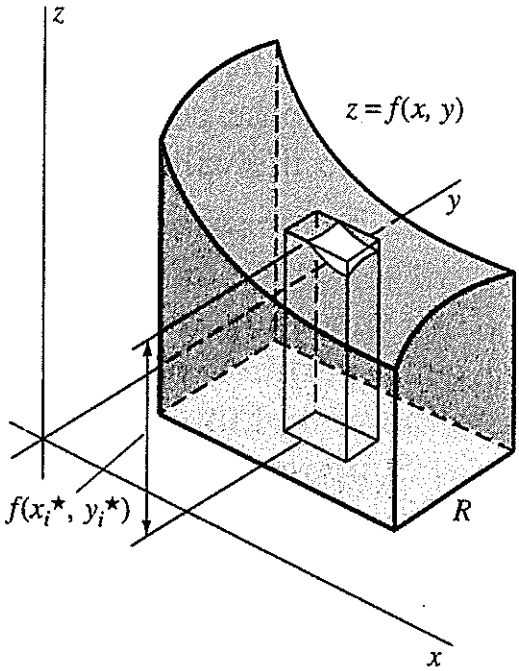
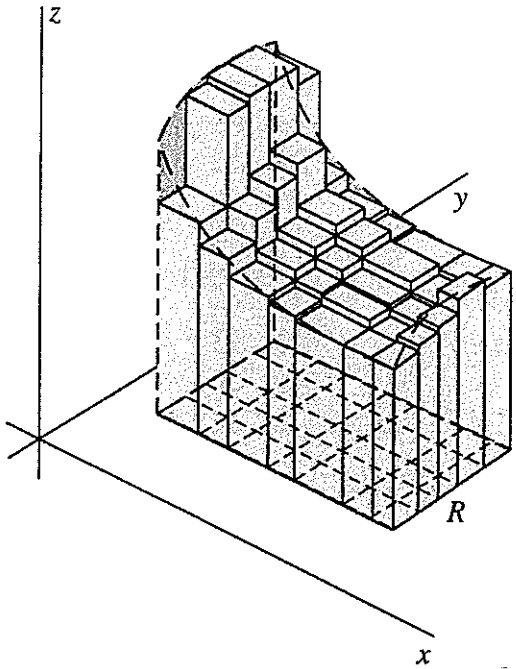
S. 940







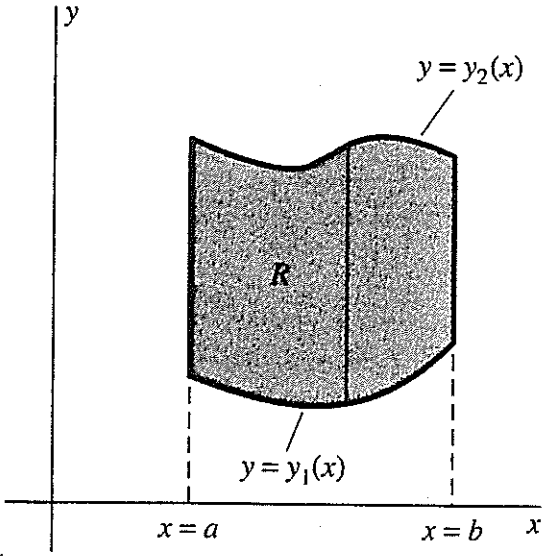


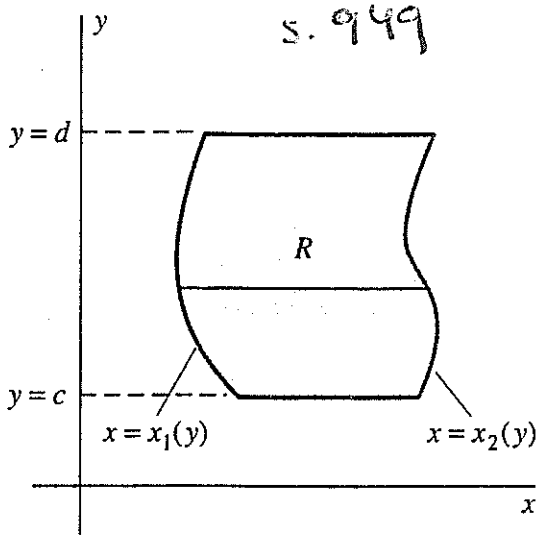
S. 942

THEOREM 1 Double Integrals as Iterated Single Integrals

Suppose that $f(x, y)$ is continuous on the rectangle $R = [a, b] \times [c, d]$. Then

$$\iint_R f(x, y)\, dA = \int_a^b \left(\int_c^d f(x, y)\, dy \right) dx = \int_c^d \left(\int_a^b f(x, y)\, dx \right) dy.$$





S. 949

THEOREM 1 Evaluation of Double Integrals

S. 950

Suppose that $f(x, y)$ is continuous on the region R. If R is the vertically simple region given in (2), then

$$\iint_R f(x, y)\, dA = \int_a^b \int_{y_1(x)}^{y_2(x)} f(x, y)\, dy\, dx.$$

If R is the horizontally simple region given in (3), then

$$\iint_R f(x, y)\, dA = \int_c^d \int_{x_1(y)}^{x_2(y)} f(x, y)\, dx\, dy.$$

Let T be the triangular region enclosed by the lines $y = 0$, $y = 2x$, and $x = 1$. Evaluate the double integral

$$\iint_T (x + y)\, dA$$

using an iterated integral with

a. y-integration first **b.** x-integration first

LHL S. 540

Eksempel 11.3.4 La D være området i $\mathbb{R}^2$ begrenset av de fire kurvene $x = -y^2$, $x = 4 - y^2$, $y = 2$ og $y = -2$. Beregn dobbelintegralet

$$\iint_D y^2\, dA.$$

Find the volume of the solid bounded above by the plane $z = y$ and below in the xy-plane by the part of the disk $x^2 + y^2 \leq 1$ in the first quadrant.

$$\iint_R cf(x, y)\, dA = c \iint_R f(x, y)\, dA,$$

s. 952

$$\iint_R [f(x, y) + g(x, y)]\, dA = \iint_R f(x, y)\, dA + \iint_R g(x, y)\, dA,$$

$$m \cdot a(R) \leqq \iint_R f(x, y)\, dA \leqq M \cdot a(R),$$

$$\iint_R f(x, y)\, dA = \iint_{R_1} f(x, y)\, dA + \iint_{R_2} f(x, y)\, dA.$$

DEFINITION Volume below $z = f(x, y)$

Suppose that the function f is continuous and nonnegative on the bounded plane region R. Then the **volume** V of the solid that lies below the surface $z = f(x, y)$ and above the region R is defined to be

$$V = \iint_R f(x, y)\, dA,$$

provided that this integral exists.

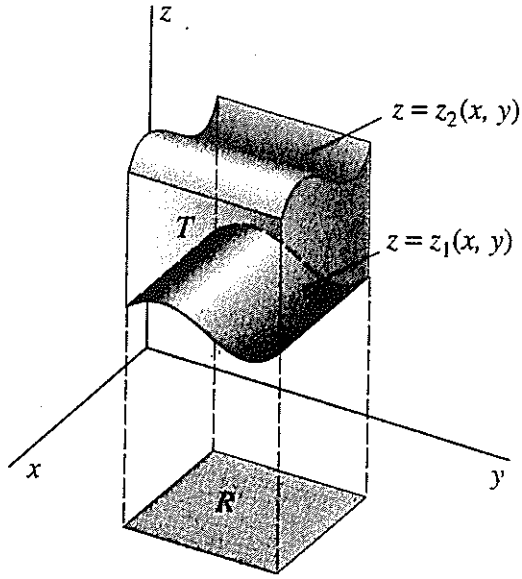
LHL s. 541

Eksempel 11.3.5 Finn volumet av legemet over området D og under grafen $z = 1 + x^2$ når $D \subseteq \mathbb{R}^2$ er området avgrenset av de to kurvene $y = x^2$ og $y = x^3$.

Find the area of the region D between $y = \cos x$ and $y = \sin x$ over the interval $0 \leq x \leq \frac{\pi}{4}$ using

a. a single integral **b.** a double integral

S. 957



$$V = \iint_R (z_{\text{top}} - z_{\text{bot}})\, dA$$

S. 958

EXAMPLE 4 Find the volume V of the solid T bounded by the planes $z = 6$ and $z = 2y$ and by the parabolic cylinders $y = x^2$ and $y = 2 - x^2$. This solid is sketched in Fig. 14.3.12.