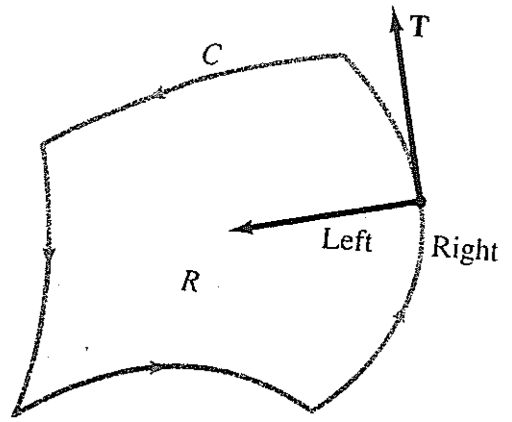
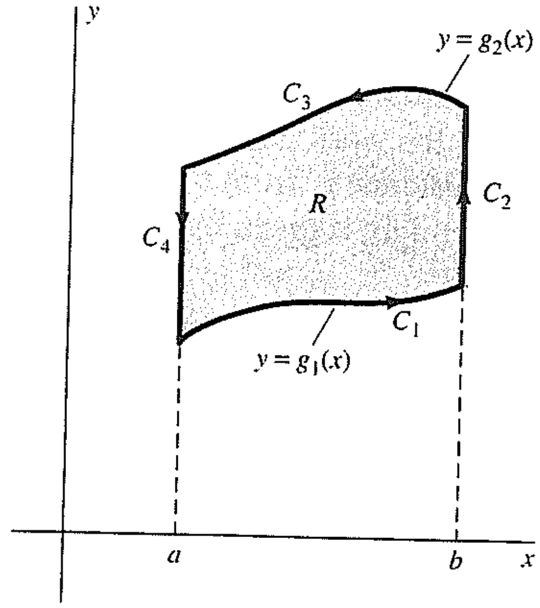




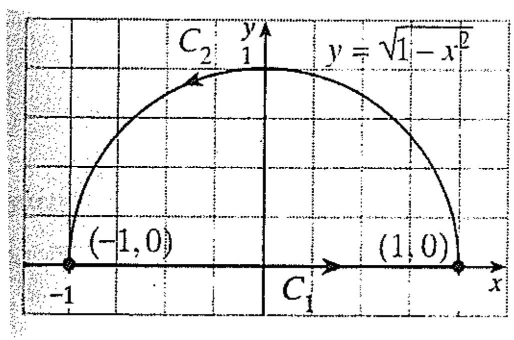
GREEN'S THEOREM

S 1038

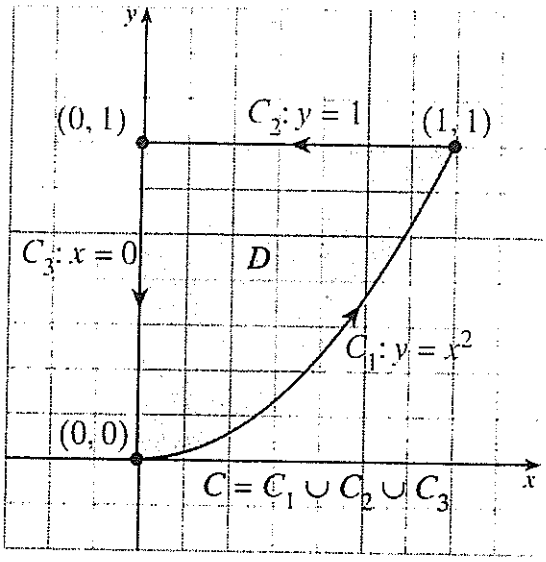
Let C be a positively oriented piecewise-smooth simple closed curve that bounds the region R in the plane. Suppose that the functions $P(x, y)$ and $Q(x, y)$ have continuous first-order partial derivatives on R. Then

$$\oint_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA.$$

Show that Green's theorem is true for the line integral $\oint_C (-y\,dx + x\,dy)$, where C is the closed path shown in Figure 13.23.



A closed path C in the plane is defined by Figure 13.24. Find the work done by an object moving along C in the force field $\mathbf{F}(x, y) = (x + xy^2)\mathbf{i} + 2(x^2 y - y^2 \sin y)\mathbf{j}$.



COROLLARY TO GREEN'S THEOREM S. 1040

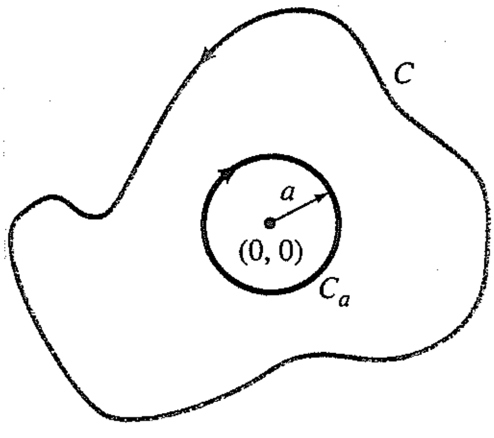
The area A of the region R bounded by the positively oriented piecewise-smooth simple closed curve C is given by

$$A = \frac{1}{2}\oint_C -y\,dx + x\,dy = -\oint_C y\,dx = \oint_C x\,dy.$$

EXAMPLE 4 Suppose that C is a positively oriented piecewise-smooth simple closed curve that encloses the origin $(0, 0)$. Show that

$$\oint_C \frac{-y\,dx + x\,dy}{x^2 + y^2} = 2\pi,$$

and also show that this integral is zero if C does *not* enclose the origin. S. 1041



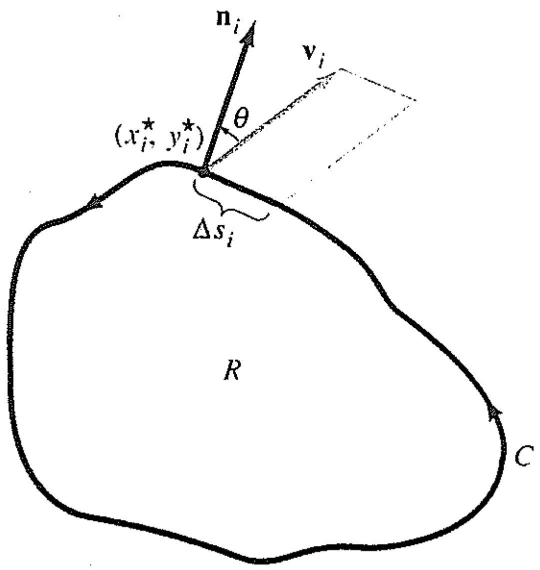


FIGURE 15.4.9 The area of the parallelogram approximates the fluid flow across Δs_i in unit time.

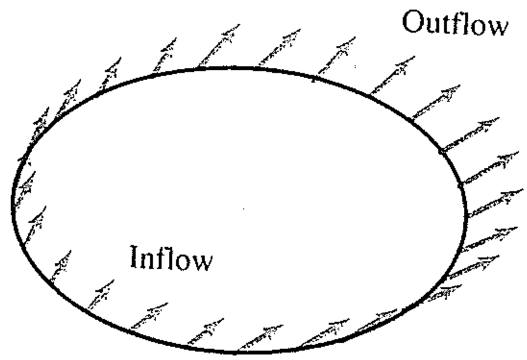


FIGURE 15.4.10 The flux Φ of the vector field **F** across the curve C is the net outflow minus the net inflow.