s. 643 – 649

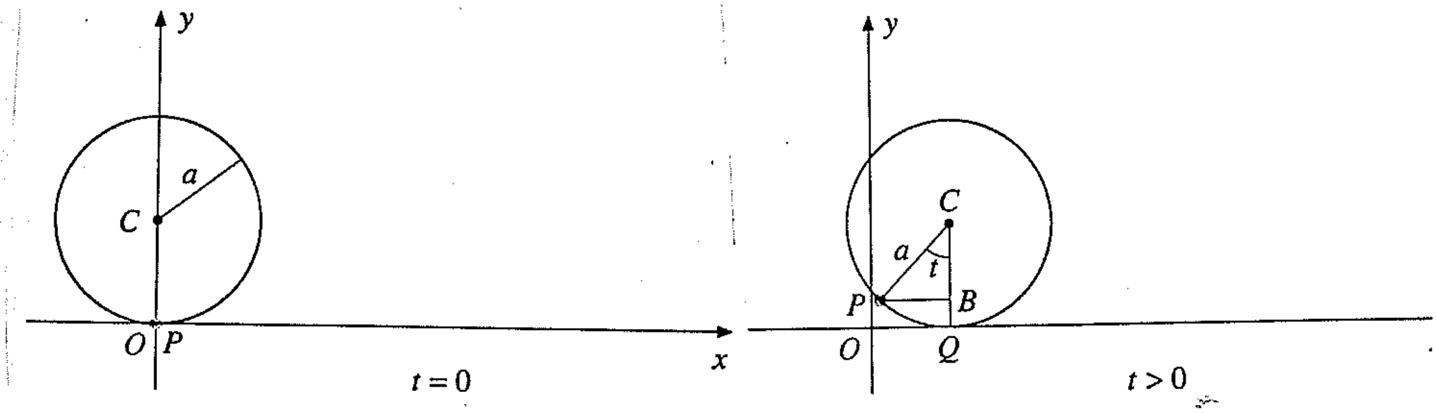
DEFINITION Parametric Curve

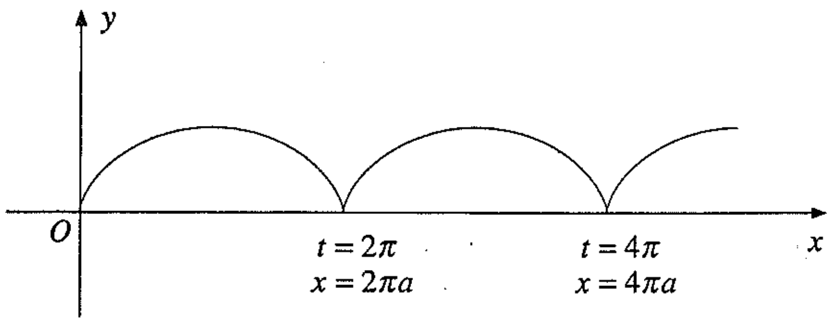
A **parametric curve** C in the plane is a pair of functions

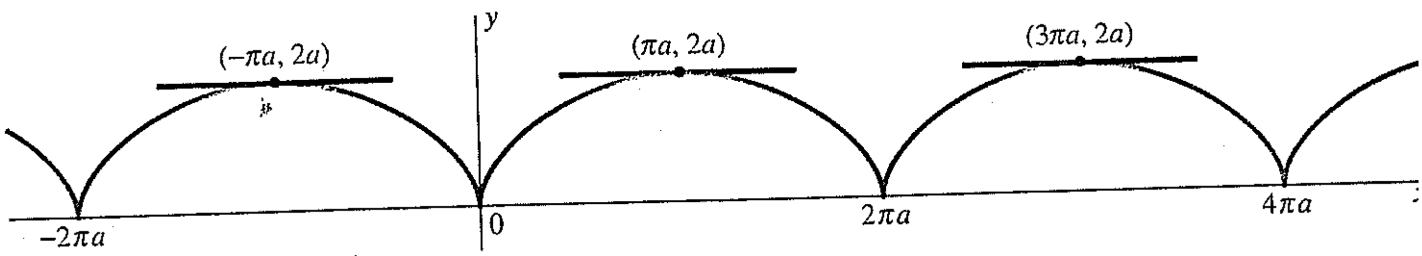
$$x = f(t), \quad y = g(t),$$

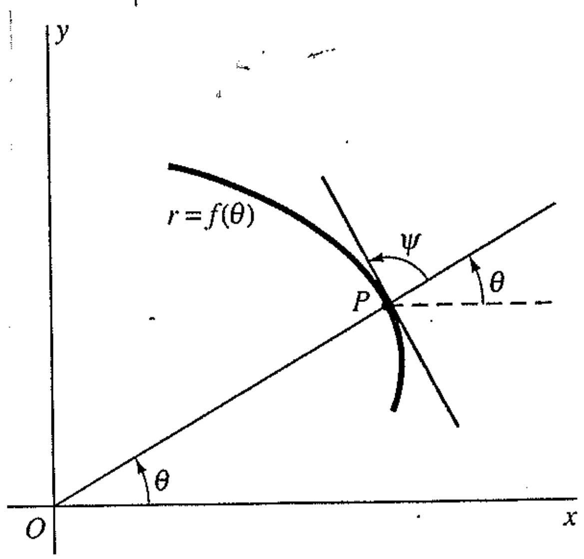
that give x and y as continuous functions of the real number t (the parameter) in some interval I.

OPPGAVE: Et hjul med radius $a > 0$ triller langs x-aksen. På hjulets ytterkant er et punkt P markert med rødt. Finn en parameterfremstilling for den kurven P beskriver når hjulet triller, dersom vi starter med P i origo.









5, 653

- The area under the curve:

$$A = \int_a^b y\,dx. \tag{1}$$

- The volume of revolution around the x-axis:

$$V_x = \int_a^b \pi y^2\,dx. \tag{2a}$$

- The volume of revolution around the y-axis:

$$V_y = \int_a^b 2\pi x y\,dx. \tag{2b}$$

- The arc length of the curve:

$$s = \int_0^s ds = \int_a^b \sqrt{1 + (dy/dx)^2}\,dx. \tag{3}$$

- The area of the surface of revolution around the x-axis:

$$S_x = \int_{x=a}^b 2\pi y\,ds. \tag{4a}$$

- The area of the surface of revolution around the y-axis:

$$S_y = \int_{x=a}^b 2\pi x\,ds. \tag{4b}$$

s. 772 - 775

DEFINITION Vector

A **vector** $\mathbf{v}$ in the Cartesian plane is an ordered pair of real numbers that has the form $\langle a, b\rangle$. We write $\mathbf{v} = \langle a, b\rangle$ and call a and b the **components** of the vector $\mathbf{v}$.

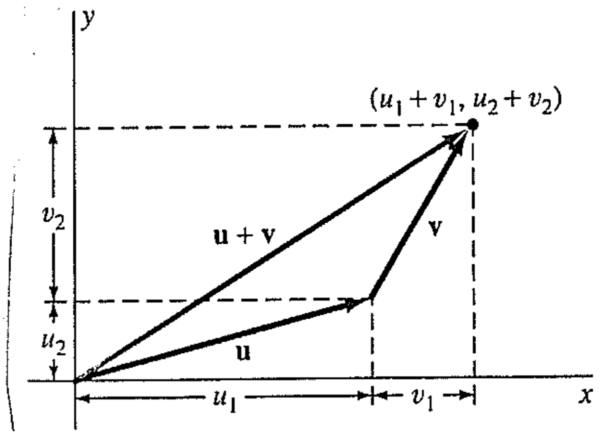
DEFINITION Equality of Vectors

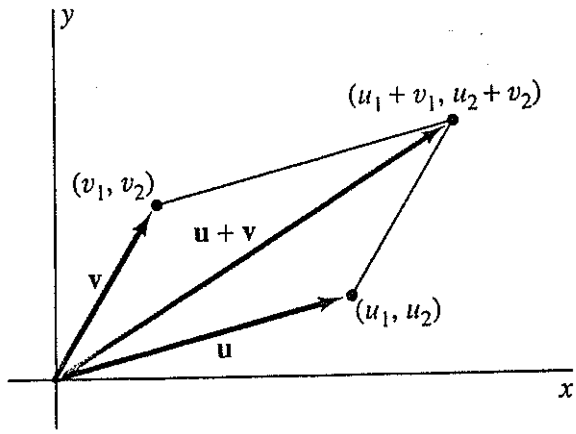
The two vectors $\mathbf{u} = \langle u_1, u_2\rangle$ and $\mathbf{v} = \langle v_1, v_2\rangle$ are **equal** provided that $u_1 = v_1$ and $u_2 = v_2$.

DEFINITION Addition of Vectors

The **sum** $\mathbf{u} + \mathbf{v}$ of the two vectors $\mathbf{u} = \langle u_1, u_2\rangle$ and $\mathbf{v} = \langle v_1, v_2\rangle$ is the vector

$$\mathbf{u} + \mathbf{v} = \langle u_1 + v_1, u_2 + v_2\rangle.$$



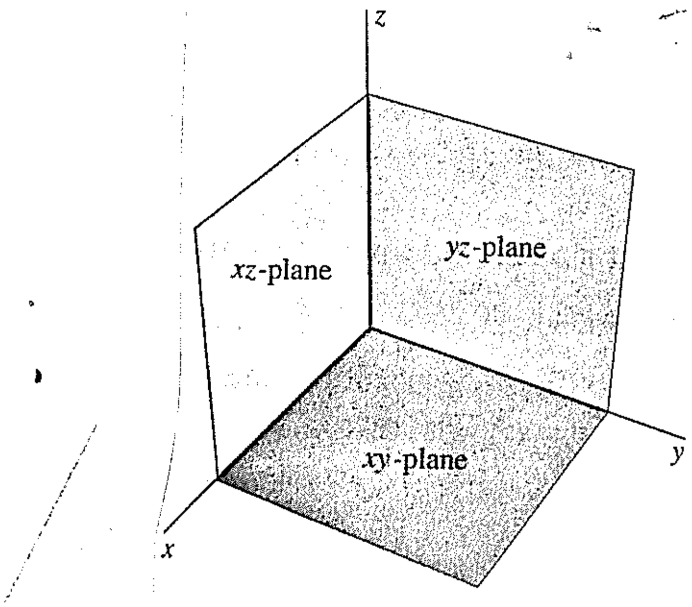


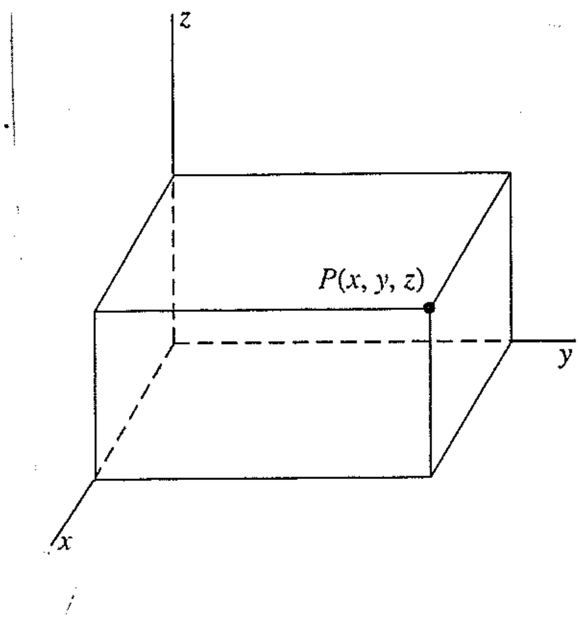
DEFINITION Multiplication of a Vector by a Scalar

If $\mathbf{u} = \langle u_1, u_2\rangle$ and c is a real number, then the **scalar multiple** $c\mathbf{u}$ is the vector

$$c\mathbf{u} = \langle cu_1, cu_2\rangle.$$

1. $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$,
2. $\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$,
3. $r(\mathbf{a} + \mathbf{b}) = r\mathbf{a} + r\mathbf{b}$,
4. $(r + s)\mathbf{a} = r\mathbf{a} + s\mathbf{a}$,
5. $(rs)\mathbf{a} = r(s\mathbf{a}) = s(r\mathbf{a})$.





The Dot Product of Two Vectors

S. 782 - 784

The **dot product** of the two vectors

$$\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k} \quad \text{and} \quad \mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$$

is the number obtained when we multiply corresponding components of **a** and **b** an[d] add the results. That is,

$$\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3. \tag{8}$$

Thus the dot product of two vectors is the *sum of the products of their correspondin[g] components.* In the case of plane vectors $\mathbf{a} = \langle a_1, a_2 \rangle$ and $\mathbf{b} = \langle b_1, b_2 \rangle$, we simpl[y] dispense with third components and write $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2$.

$$\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2,$$

$$\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a},$$

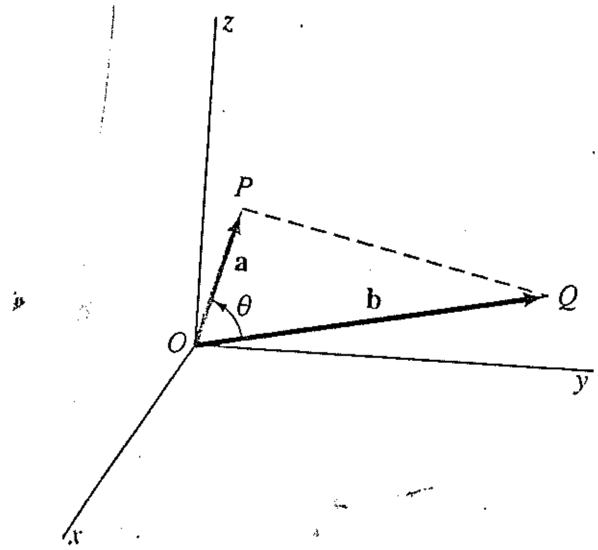
$$\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c},$$

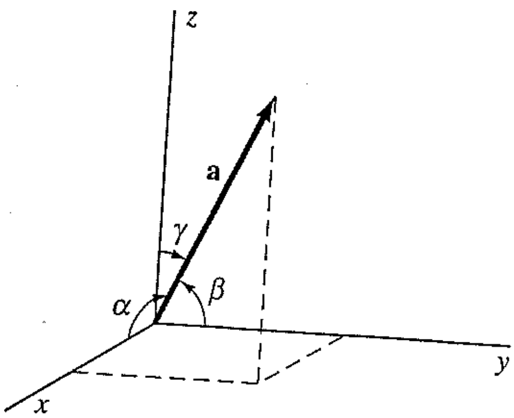
$$(r\mathbf{a}) \cdot \mathbf{b} = r(\mathbf{a} \cdot \mathbf{b}) = \mathbf{a} \cdot (r\mathbf{b}).$$

THEOREM 1 Interpretation of the Dot Product

If θ is the angle between the vectors **a** and **b**, then

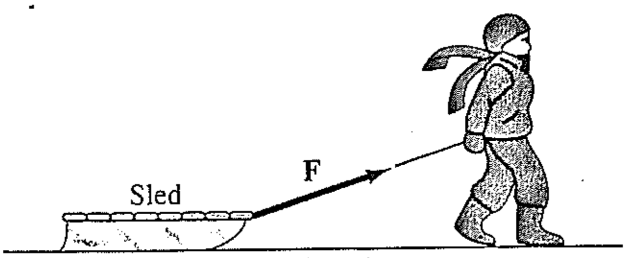
$$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}|\,|\mathbf{b}| \cos\theta.$$

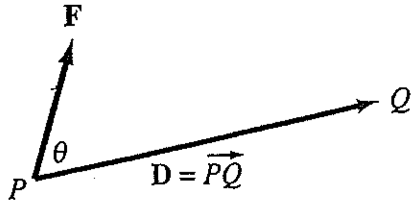




s. 785 - 791

OPPG: DEKOMPONER VEKTOREN $\bar{a} = \langle 3, -6, 1 \rangle$ I EN VEKTOR PARALLELL TIL OG EN VINKELRETT PÅ $\bar{b} = \langle 2, 1, 1 \rangle$.

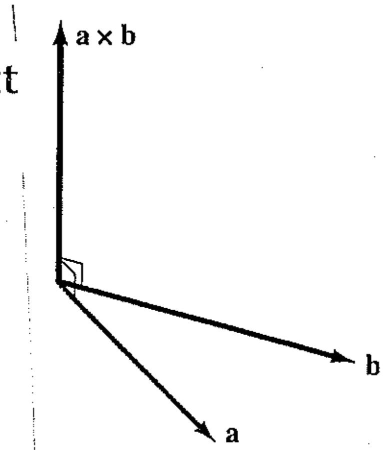




The **cross product** (or **vector product**) of the vectors $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$ is defined algebraically by the formula

$$\mathbf{a} \times \mathbf{b} = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle.$$

THEOREM 1 Perpendicularity of the Cross Product
The cross product $\mathbf{a} \times \mathbf{b}$ is perpendicular both to $\mathbf{a}$ and to $\mathbf{b}$.



THEOREM 2 Length of the Cross Product
Let θ be the angle between the nonzero vectors $\mathbf{a}$ and $\mathbf{b}$ (measured so that $0 \leqq \theta \leqq \pi$). Then

$$|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}|\,|\mathbf{b}| \sin \theta.$$

COROLLARY Parallel Vectors
Two nonzero vectors $\mathbf{a}$ and $\mathbf{b}$ are parallel ($\theta = 0$ or $\theta = \pi$) if and only if $\mathbf{a} \times \mathbf{b} = \mathbf{0}$

Eks: FINN AREALET AV TREKANTEN I XY-PLANET MED HJØRNER I (0,0), (1,1) OG (-3;5).

$$\mathbf{i} \times \mathbf{j} = \mathbf{k}, \quad \mathbf{j} \times \mathbf{k} = \mathbf{i}, \quad \text{and} \quad \mathbf{k} \times \mathbf{i} = \mathbf{j}.$$

$$\mathbf{j} \times \mathbf{i} = -\mathbf{k}, \quad \mathbf{k} \times \mathbf{j} = -\mathbf{i}, \quad \text{and} \quad \mathbf{i} \times \mathbf{k} = -\mathbf{j}.$$

$$\mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = \mathbf{0},$$

THEOREM 3 Algebraic Properties of the Cross Product

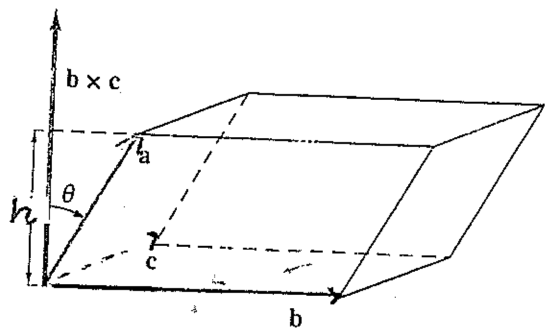
If **a**, **b**, and **c** are vectors and k is a real number, then

1. $\mathbf{a} \times \mathbf{b} = -(\mathbf{b} \times \mathbf{a})$;
2. $(k\mathbf{a}) \times \mathbf{b} = \mathbf{a} \times (k\mathbf{b}) = k(\mathbf{a} \times \mathbf{b})$;
3. $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) + (\mathbf{a} \times \mathbf{c})$;
4. $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$;
5. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$.

THEOREM 4 Scalar Triple Products and Volume

The volume V of the parallelepiped determined by the vectors **a**, **b**, and **c** is the absolute value of the scalar triple product $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})$; that is,

$$V = |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|.$$



s. 792-793