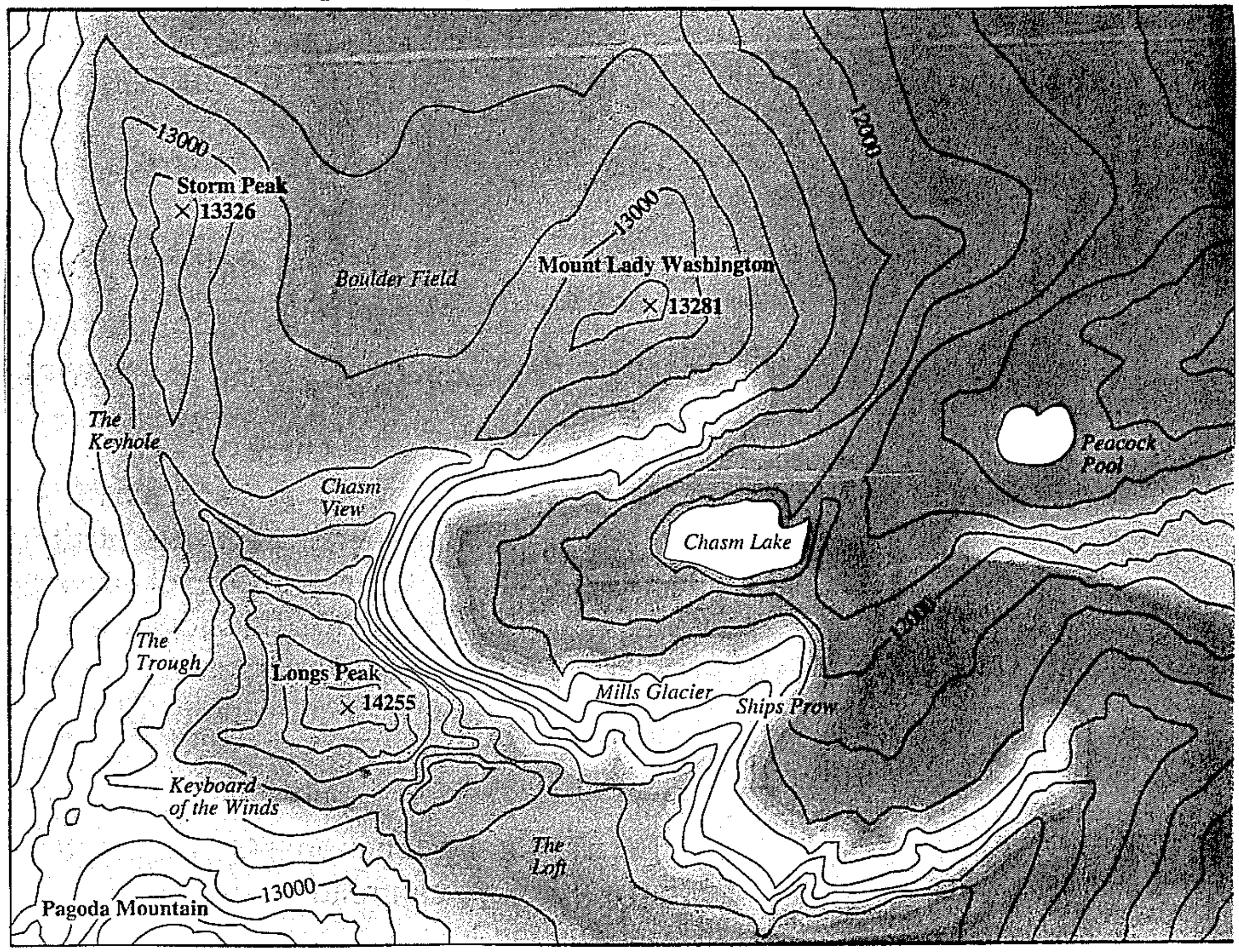
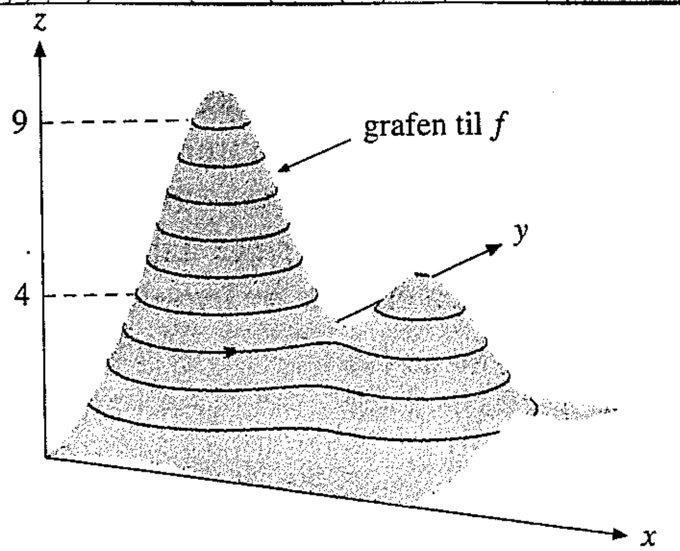
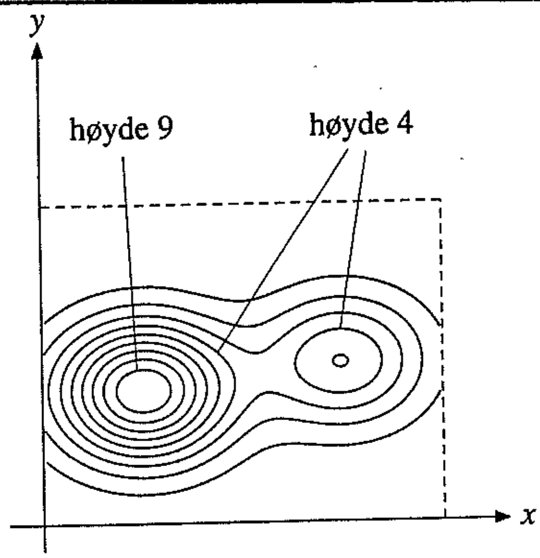
DEFINITION Functions of Two or Three Variables

A **function of two variables,** defined on the **domain** D in the plane, is a rule f th
associates with each point (x, y) in D a unique real number, denoted by $f(x, y$
A **function of three variables,** defined on the **domain** D in space, is a rule f th
associates with each point (x, y, z) in D a unique real number $f(x, y, z)$.







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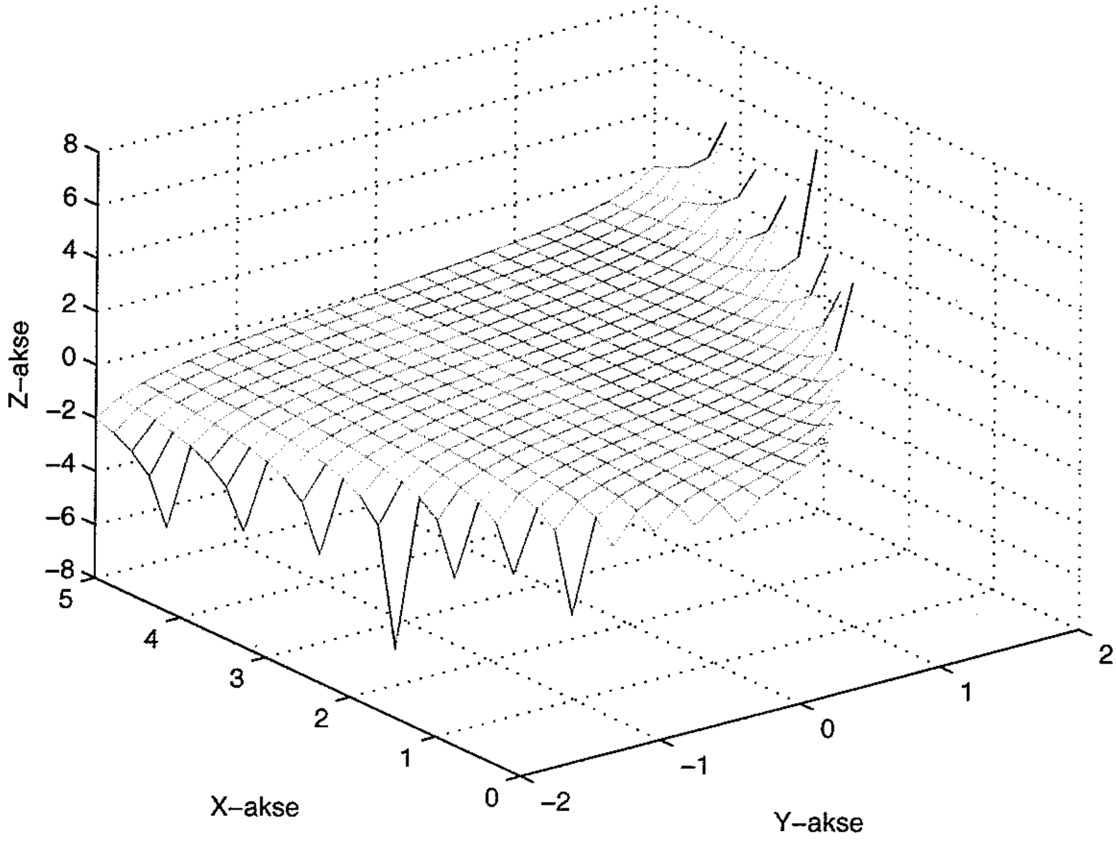
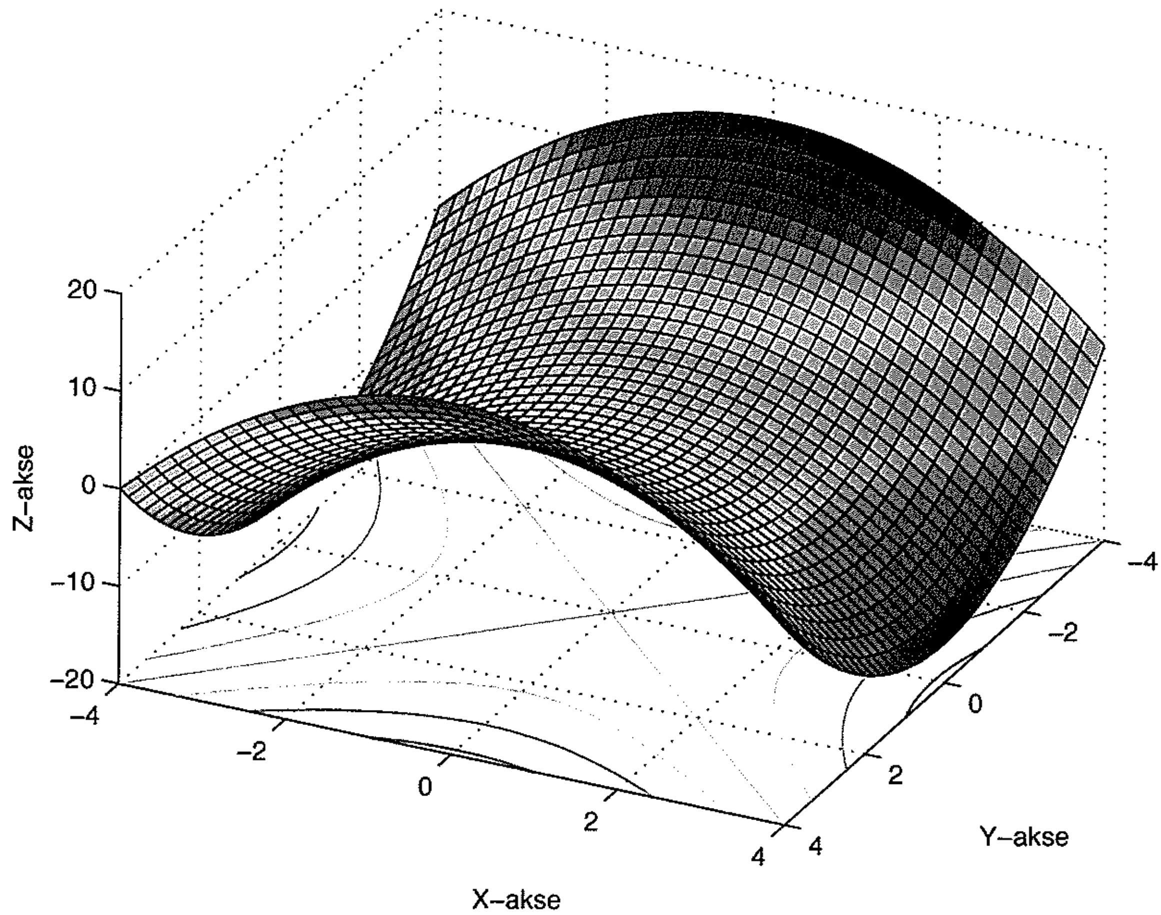
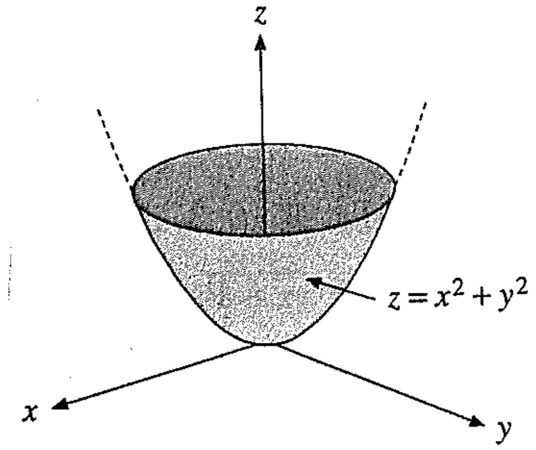
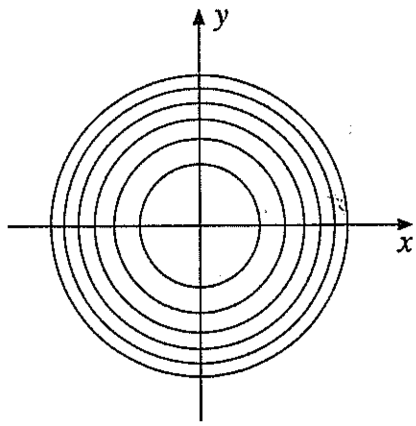


Z-akse
8
6
4
2
0
-2
-4
-6
-8
X-akse
5
4
3
2
1
0
Y-akse
-2
-1
0
1
2


20
10
0
−10
−20
Z−akse
−4
−2
0
2
4
X−akse
−4
−2
0
2
4
Y−akse

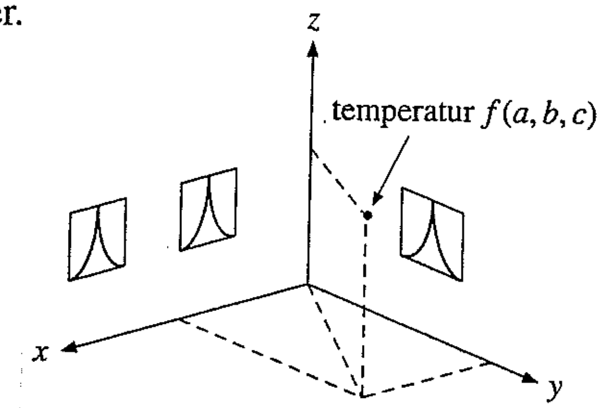




Eksempel 10.1.4 Temperaturen (målt i grader Celsius) i salen i figur 10.1.6 er gitt ved

$$f(x, y, z) = 20 - x + \tfrac{1}{8}yz \qquad \text{for} \quad 0 \le x \le 10, 0 \le y \le 20, 0 \le z \le 3,$$

der x, y, og z er gitt i meter.



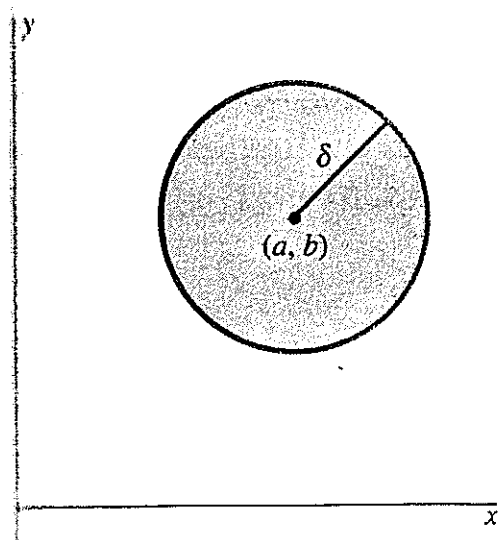
DEFINITION The Limit of $f(x, y)$

We say that the **limit of** $f(x, y)$ **as** (x, y) **approaches** (a, b) **is** L provided that fo every number $\epsilon > 0$, there exists a number $\delta > 0$ with the following property: (x, y) is a point of the domain of f such that if

$$0 < \sqrt{(x-a)^2 + (y-b)^2} < \delta,$$

then it follows that

$$|f(x, y) - L| < \epsilon.$$



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$$\lim_{(x,y)\to(a,b)} f(x,y) = L \quad \text{and} \quad \lim_{(x,y)\to(a,b)} g(x,y) = M,$$

then the sum, product, and quotient laws for limits are these:

$$\lim_{(x,y)\to(a,b)} [f(x,y) + g(x,y)] = L + M,$$

$$\lim_{(x,y)\to(a,b)} [f(x,y) \cdot g(x,y)] = L \cdot M,$$

$$\text{and} \quad \lim_{(x,y)\to(a,b)} \frac{f(x,y)}{g(x,y)} = \frac{L}{M} \quad \text{if } M \neq 0.$$

Grenselover for funksjoner av flere variable 10.2.2 La f og g være to funksjoner av n variable, og la h være en funksjon av én variabel. Hvis $\lim_{\mathbf{x}\to\mathbf{a}} f(\mathbf{x}) = r$ og $\lim_{\mathbf{x}\to\mathbf{a}} g(\mathbf{x}) = s$, så er

A. $\lim_{\mathbf{x}\to\mathbf{a}} [f(\mathbf{x}) + g(\mathbf{x})] = r + s$

B. $\lim_{\mathbf{x}\to\mathbf{a}} [f(\mathbf{x}) - g(\mathbf{x})] = r - s$

C. $\lim_{\mathbf{x}\to\mathbf{a}} f(\mathbf{x}) \cdot g(\mathbf{x}) = r \cdot s$

D. $\lim_{\mathbf{x}\to\mathbf{a}} \frac{f(\mathbf{x})}{g(\mathbf{x})} = \frac{r}{s}$ hvis $s \neq 0$

E. $\lim_{\mathbf{x}\to\mathbf{a}} h(g(\mathbf{x})) = h(s)$ hvis h er kontinuerlig i s

DEFINITION Partial Derivatives

The **partial derivatives (with respect to x and with respect to y)** of the functic $f(x, y)$ are the two functions defined by

$$f_x(x,y) = \lim_{h\to 0} \frac{f(x+h,y) - f(x,y)}{h},$$

$$f_y(x,y) = \lim_{k\to 0} \frac{f(x,y+k) - f(x,y)}{k}$$

whenever these limits exist.

Notation for Partial Derivatives

If $z=f(x, y)$, then we may express its partial derivatives with respect to x and respectively, in these forms:

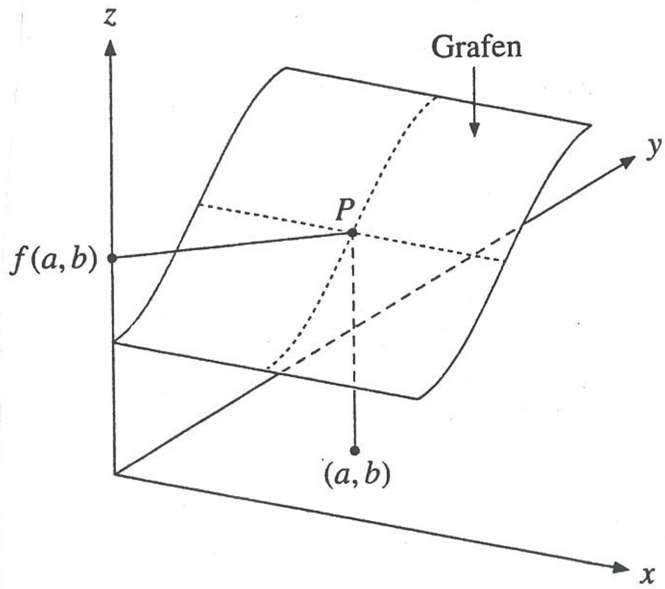
$$\frac{\partial z}{\partial x}=\frac{\partial f}{\partial x}=f_x(x, y)=\frac{\partial}{\partial x}f(x, y)=D_x[f(x, y)]=D_1[f(x, y)],$$

$$\frac{\partial z}{\partial y}=\frac{\partial f}{\partial y}=f_y(x, y)=\frac{\partial}{\partial y}f(x, y)=D_y[f(x, y)]=D_2[f(x, y)].$$

EXAMPLE 4 The volume V (in cubic centimeters) of 1 mole (mol) of an ideal gas is given by

$$V=\frac{(82.06)T}{p},$$

where p is the pressure (in atmospheres) and T is the absolute temperature (in kelvins (K), where $K={}^\circ C+273$). Find the rates of change of the volume of 1 mol of an ideal gas with respect to pressure and with respect to temperature when $T=300$ K and $p=5$ atm.



Eksempel 10.3.4 Et landskap kan beskrives som grafen til $f(x, y) = x^2 + 3\sin(xy\pi)$ i og nær punktet $\left(1, \frac{1}{2}, f\left(1, \frac{1}{2}\right)\right) = \left(1, \frac{1}{2}, 4\right)$. En fjellklatrer beveger seg rett østover gjennom punktet. Hvor bratt er fjellklatrerens trasé i punktet når y-aksen peker rett nordover og x-aksen rett østover?

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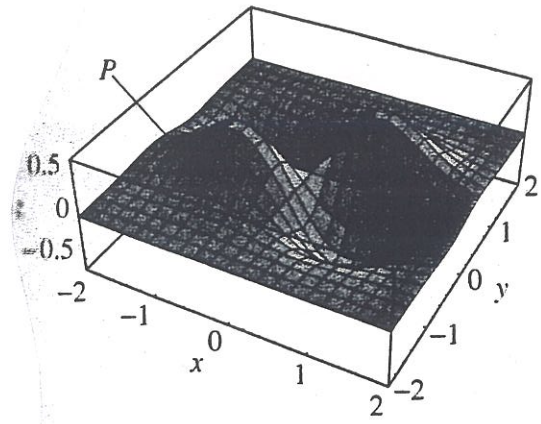
EXAMPLE 5 Suppose that the graph $z = 5xy\exp(-x^2 - 2y^2)$ in Fig. 13.4.7 represents a terrain featuring two peaks (hills, actually) and two pits. With all distances measured in miles, z is the altitude above the point (x, y) at sea level in the xy-plane



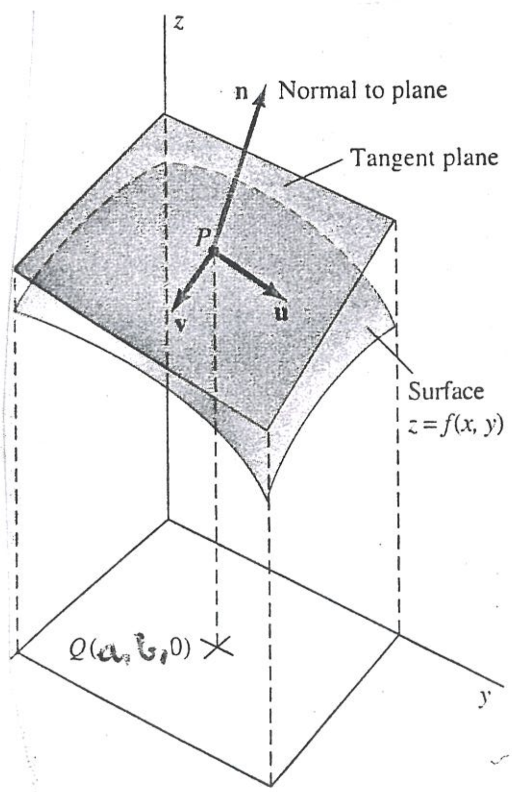
DEFINITION Plane Tangent to $z = f(x, y)$

Suppose that the function $f(x, y)$ has continuous partial derivatives on a circular disk centered at the point (a, b). Then the **plane tangent** to the surface $z = f(x, y)$ at the point $P(a, b, f(a, b))$ is the plane through P that contains the lines tangent at P to the two curves

$$z = f(x, b), \quad y = b \quad (x\text{-curve})$$

and

$$z = f(a, y), \quad x = a \quad (y\text{-curve}).$$



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Eksempel 10.4.7 Finn likningen for tangentplanet til flaten $z = f(x, y) = x^2 + y^2$ i punktet $(1, 0, 1)$ dersom dette tangentplanet finnes.

$$\frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(x+h, y, z) - f(x, y, z)}{h},$$

$$\frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(x, y+h, z) - f(x, y, z)}{h},$$

$$\frac{\partial f}{\partial z} = \lim_{h \to 0} \frac{f(x, y, z+h) - f(x, y, z)}{h}.$$

$$(f_x)_x = f_{xx} = \frac{\partial f_x}{\partial x} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2},$$

$$(f_x)_y = f_{xy} = \frac{\partial f_x}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \, \partial x},$$

$$(f_y)_x = f_{yx} = \frac{\partial f_y}{\partial x} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x \, \partial y},$$

$$(f_y)_y = f_{yy} = \frac{\partial f_y}{\partial y} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2}.$$

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