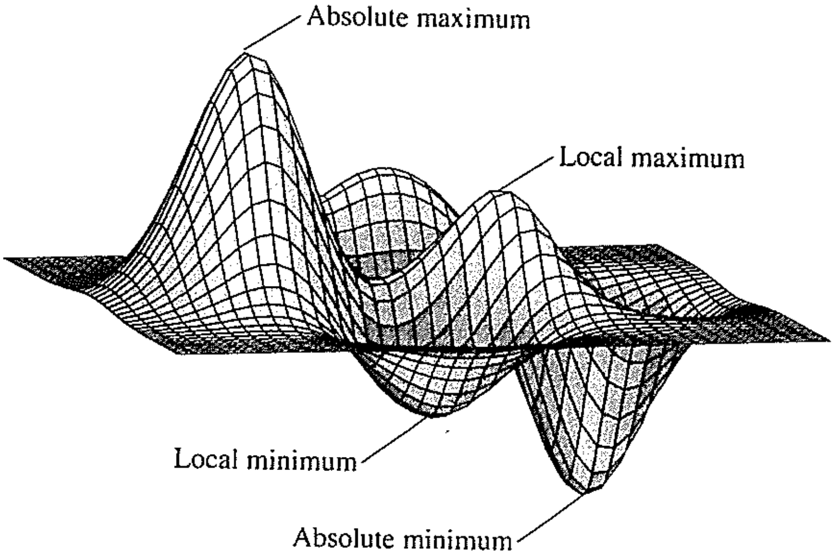
THEOREM 1 Existence of Extreme Values
S. 879

Suppose that the function f is continuous on the region R that consists of the points on and within a simple closed curve C in the plane. Then f attains an absolute maximum value at some point (a, b) of R and attains an absolute minimum value at some point (c, d) of R.



S. 880

THEOREM 2 Necessary Conditions for Local Extrema

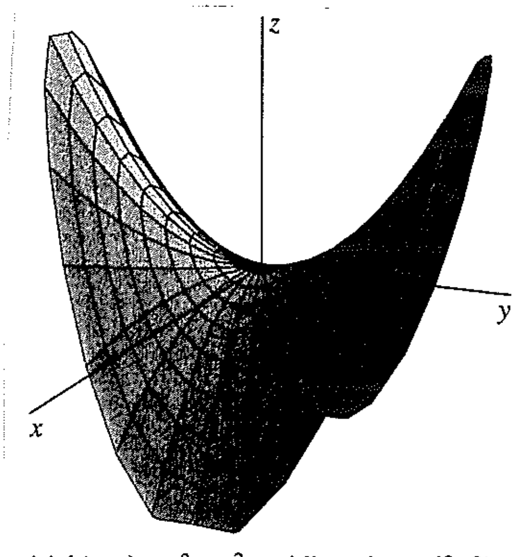
Suppose that $f(x, y)$ attains a local maximum value or a local minimum value at the point (a, b) and that both the partial derivatives $f_x(a, b)$ and $f_y(a, b)$ exist. Then

$$f_x(a, b) = 0 = f_y(a, b).$$

THEOREM 3 Types of Absolute Extrema
S. 881

Suppose that f is continuous on the plane region R consisting of the points on and within a simple closed curve C. If $f(a, b)$ is either the absolute maximum or the absolute minimum value of $f(x, y)$ on R, then (a, b) is either

1. An interior point of R at which
$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = 0,$$
2. An interior point of R where not both partial derivatives exist, or
3. A point of the boundary curve C of R.



(c) $h(x, y) = y^2 - x^2$, saddle point at $(0, 0)$

Finn mulige maksimum- og minimumspunkt for funksjonen $f(x,y) = x^2 + xy + y^2$ definert på halvsirkelen $x^2 + y^2 \leq 1$ og $y \geq 0$.

Finn alle punkt på flata $y^2 = 4 + xz$ som er nærmest origo.

s. 480 LHL

Eksempel 10.4.4 Et glassverk får i oppdrag å produsere måleglass i form av en rett sylinder med indre diameter 10 cm og indre høyde $2/\pi$ m. Glassverket kan produsere måleglassene med en nøyaktighet på ± 2 mm på de to målene . Hvor stor blir nøyaktigheten av volumet?

s. 891

EXAMPLE 3 In Example 4 of Section 13.4, we considered 1 mole of an ideal gas—its volume V in cubic centimeters given in terms of its pressure p in atmospheres and temperature T in kelvins by the formula $V = (82.06)T/p$. Approximate the change in V when p is increased from 5 atm to 5.2 atm and T is increased from 300 K to 310 K.

s. 893

THEOREM Linear Approximation

Suppose that the function $f(\mathbf{x})$ of n variables has continuous first-order partial derivatives in a region that contains the neighborhood $|\mathbf{x} - \mathbf{a}| < r$ consisting of all points $\mathbf{x}$ at distance less than r from the fixed point $\mathbf{a}$. If $\mathbf{a} + \mathbf{h}$ lies in this neighborhood, then

$$f(\mathbf{a} + \mathbf{h}) = f(\mathbf{a}) + \nabla f(\mathbf{a}) \cdot \mathbf{h} + \boldsymbol{\epsilon}(\mathbf{h}) \cdot \mathbf{h} \qquad \textbf{(15)}$$

where $\boldsymbol{\epsilon}(\mathbf{h}) = \langle \epsilon_1(\mathbf{h}), \epsilon_2(\mathbf{h}), \ldots, \epsilon_n(\mathbf{h}) \rangle$ is a vector such that each element $\epsilon_i(\mathbf{h})$ approaches zero as $\mathbf{h} \to \mathbf{0}$.

THEOREM 1 The Chain Rule

Suppose that $w = f(x, y)$ has continuous first-order partial derivatives and that $x = g(t)$ and $y = h(t)$ are differentiable functions. Then w is a differentiable function of t, and

$$\frac{dw}{dt} = \frac{\partial w}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial w}{\partial y} \cdot \frac{dy}{dt}.$$

s. 897

s. 485 LHL

Eksempel 10.5.3 En humle stiger til værs langs helixen

$$x = \cos t, \quad y = \sin t, \quad z = t/2 \qquad \text{for} \quad 0 \le t \le \infty$$

i en sone der temperaturen er gitt ved $x^2 + y^2 - 2z$. Finn hvor rask temperaturendring humlen opplever som funksjon av tiden t.

THEOREM 2 The General Chain Rule s. 899

Suppose that w is a function of the variables $x_1, x_2, \ldots, x_m$ and that each of these is a function of the variables $t_1, t_2, \ldots, t_n$. If all these functions have continuous first-order partial derivatives, then

$$\frac{\partial w}{\partial t_i} = \frac{\partial w}{\partial x_1} \cdot \frac{\partial x_1}{\partial t_i} + \frac{\partial w}{\partial x_2} \cdot \frac{\partial x_2}{\partial t_i} + \cdots + \frac{\partial w}{\partial x_m} \cdot \frac{\partial x_m}{\partial t_i}$$

for each i, $1 \le i \le n$.

EXAMPLE 5 Let $w = f(x, y)$ where x and y are given in polar coordinates by the equations $x = r\cos\theta$ and $y = r\sin\theta$. Calculate

$$\frac{\partial w}{\partial r}, \quad \frac{\partial w}{\partial \theta}, \quad \text{and} \quad \frac{\partial^2 w}{\partial r^2}$$

in terms of r, θ, and the partial derivatives of w with respect to x and y (Fig. 13.7.5).

THEOREM 3 Implicit Function Theorem s. 901

Suppose that the function $F(x_1, x_2, \ldots, x_n, z)$ is continuously differentiable near the point $(\mathbf{a}, b) = (a_1, a_2, \ldots, n, b)$ at which $F(\mathbf{a}, b) = 0$ and $D_z F(\mathbf{a}, b) \neq 0$. Then there exists a continuously differentiable function $z = g(x_1, x_2, \ldots, x_n)$ such that $g(\mathbf{a}) = b$ and $F(\mathbf{x}, g(\mathbf{x})) = 0$ for $\mathbf{x}$ near $\mathbf{a}$.

Funksjonen $F(x, y, z) = x^3 + y^3 + z^3 + 4xyz - 28 = 0$ definerer en flate i rommet. Finn tangentplanet for $x = 1$ og $y = 0$.