

Maple-worksheet som som viser noen løsninger av svingeligningen uten pådrag.
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Definerer først diff.ligningen for systemet og finner generell løsning når massen $m=1$, fjærkonstanten $k=1$.

Klossen står stille i likevektsposisjon og vi gir den en impuls som gir hastighet 1. [Altså $x(0)=0$ og $v(0)=1$]

```
> ode1 := m*diff(x(t), t, t) + c*diff(x(t), t) + k*x(t);
```

$$ode1 := m \left(\frac{d^2}{dt^2} x(t) \right) + c \left(\frac{d}{dt} x(t) \right) + k x(t)$$

```
> m := 1: k := 1:
```

```
> dsolve( {ode1, x(0)=0, D(x)(0)=1} );
```

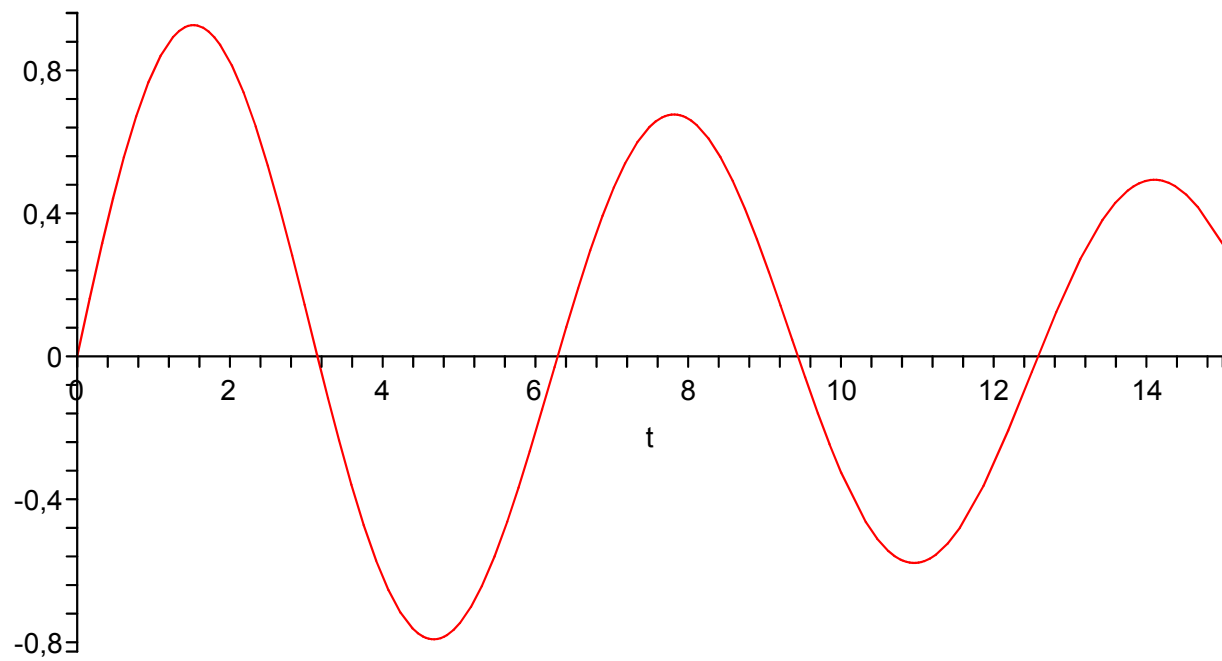
$$x(t) = \frac{e^{\left(\left(-\frac{1}{2}c + \frac{1}{2}\sqrt{c^2 - 4} \right) t \right)}}{\sqrt{c^2 - 4}} - \frac{e^{\left(\left(-\frac{1}{2}c - \frac{1}{2}\sqrt{c^2 - 4} \right) t \right)}}{\sqrt{c^2 - 4}}$$

Finner og plotter løsning for $c=0.1$ (Underdempet system)

```
> c := 0.1: dsolve( {ode1, x(0)=0, D(x)(0)=1} );
```

$$x(t) = \frac{20}{399} \sqrt{399} e^{\left(-\frac{1}{20} t \right)} \sin\left(\frac{1}{20} \sqrt{399} t \right)$$

```
> L1 := 20/399*399^(1/2)*exp(-1/20*t)*sin(1/20*399^(1/2)*t):  
plot(L1, t=0..15, color=red);
```

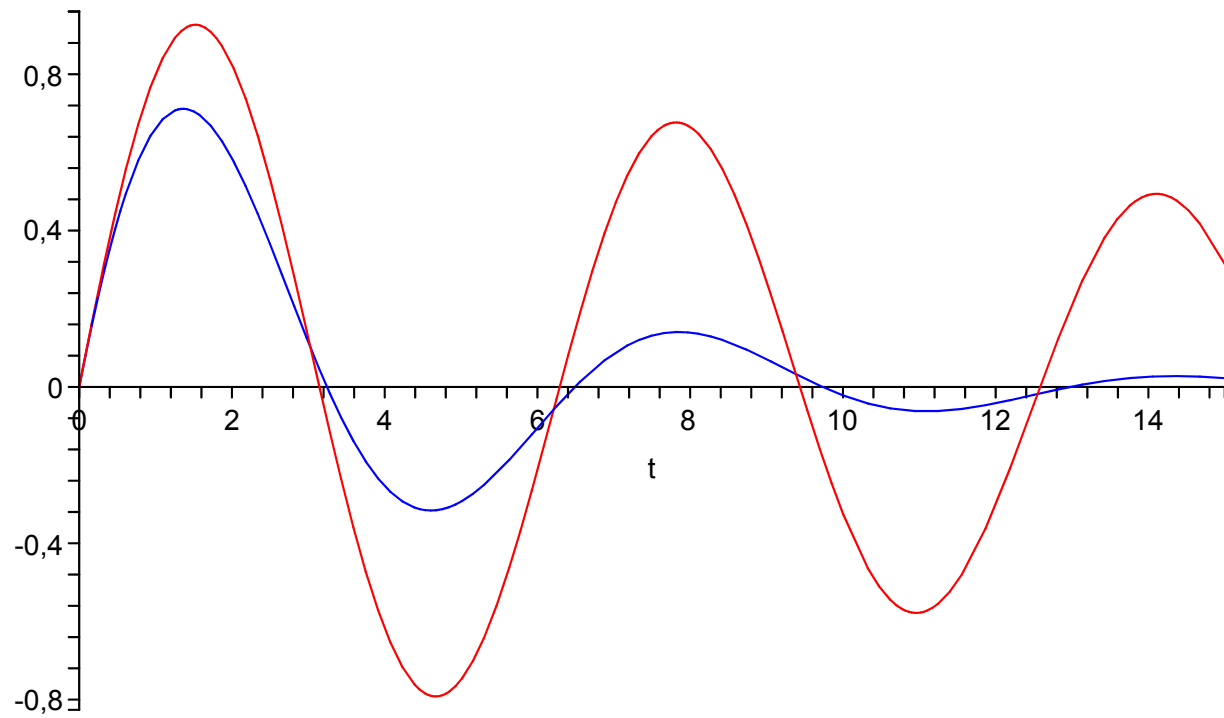


Finner og plotter løsning for $c=0.5$ (Underdempet system)

```
> c := 0.5: dsolve( {ode1,x(0)=0,D(x)(0)=1} );
```

$$x(t) = \frac{4}{15} \sqrt{15} e^{\left(-\frac{1}{4}t\right)} \sin\left(\frac{1}{4}\sqrt{15}t\right)$$

```
> L2 := 4/15*15^(1/2)*exp(-1/4*t)*sin(1/4*15^(1/2)*t):
plot([L1,L2],t=0..15,color=[red,blue]);
```



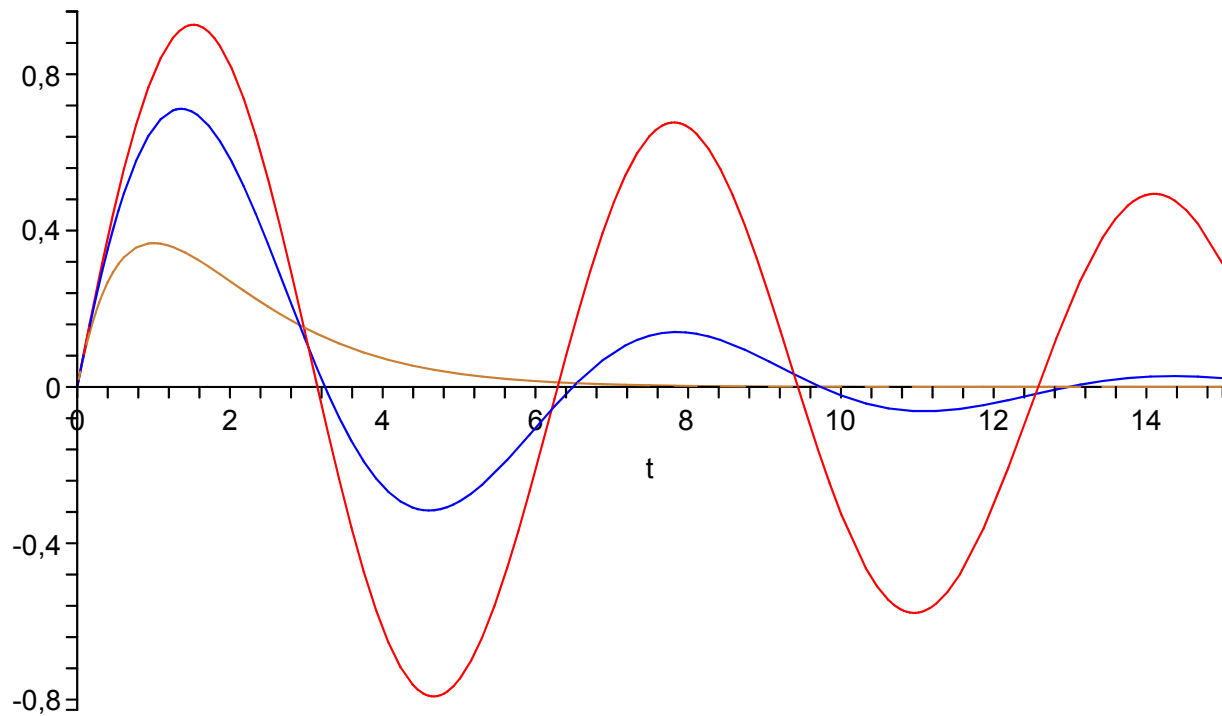
Finner og plotter løsning for $c=2$ (Kritisk demping)

```
> c := 2; dsolve( {ode1, x(0)=0, D(x)(0)=1} );
      c := 2
```

$$x(t) = e^{(-t)} t$$

```
> L3 := exp(-t)*t; plot([L1,L2,L3], t=0..15, color=[red,blue,gold]);
```

$$L3 := e^{(-t)} t$$



Finner og plotter løsning for $c=10$ (Overdamping)

```
> c := 10: dsolve( {ode1, x(0)=0, D(x)(0)=1} );
```

$$x(t) = \frac{1}{24} \sqrt{6} e^{((-5+2\sqrt{6})t)} - \frac{1}{24} \sqrt{6} e^{(-5+2\sqrt{6})t}$$

```
> L4 := 1/6*3^(1/2)*exp((-2+3^(1/2))*t)-1/6*3^(1/2)*exp(-(2+3^(1/2))*t);
plot([L3,L4], t=0..15, color=[gold,green]);
```

$$L4 := \frac{1}{6} \sqrt{3} e^{((-2+\sqrt{3})t)} - \frac{1}{6} \sqrt{3} e^{(-2+\sqrt{3})t}$$

