

Eks: Finn og plott fjerderøttene til -4
(fra forelesningen mandag 28/8 kl 12.15–14)

$$\sqrt[4]{-4} = ? \quad \text{dvs.} \quad w^4 = -4, \quad w = ?$$

Polarform:

$$w = R(\cos \phi + i \sin \phi) = Re^{i\phi}$$

$$-4 = 4(\cos \pi + i \sin \pi) = 4e^{i\pi} \quad (|-4| = 4, \text{ Arg}(-4) = \pi \quad (\text{tegn figur}))$$

$$w^4 = -4 \iff R^4 e^{i(4\phi)} = 4e^{i\pi} \iff \begin{cases} R^4 = 4 \\ 4\phi = \pi + 2k\pi, \quad k \in \mathbb{Z} \end{cases}$$

$$R = \sqrt[4]{4} = \sqrt{2}, \quad \phi = \frac{\pi + 2k\pi}{4} = \frac{\pi}{4} + \frac{k\pi}{2}$$

$$w_k = \sqrt{2} e^{i(\pi/4 + k\pi/2)}, \quad k = 0, 1, 2, 3$$

$$w_0 = \sqrt{2} e^{i\pi/4} = \sqrt{2} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right) = 1 + i \quad (\text{hovedverdien})$$

$$w_1 = \sqrt{2} e^{i(\pi/4 + \pi/2)} = \sqrt{2} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right) = -1 + i$$

$$w_2 = \sqrt{2} e^{i(\pi/4 + 2\pi/2)} = \sqrt{2} \left(\cos \frac{5\pi}{4} + i \sin \frac{5\pi}{4} \right) = -1 - i$$

$$w_3 = \sqrt{2} e^{i(\pi/4 + 3\pi/2)} = \sqrt{2} \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right) = 1 - i$$

Kontroll:

$$w_0^4 = (1 + i)^4 = \left((1 + i)^2 \right)^2 = (1 + 2i + i^2)^2 = (2i)^2 = 4i^2 = -4$$

$$w_1 = \sqrt{2} e^{i\pi/4} e^{i\pi/2} = w_0 i = (1 + i)i = -1 + i$$

$$w_2 = w_1 i = (-1 + i)i = -1 - i, \quad w_3 = w_2 i = (-1 - i)i = 1 - i$$

