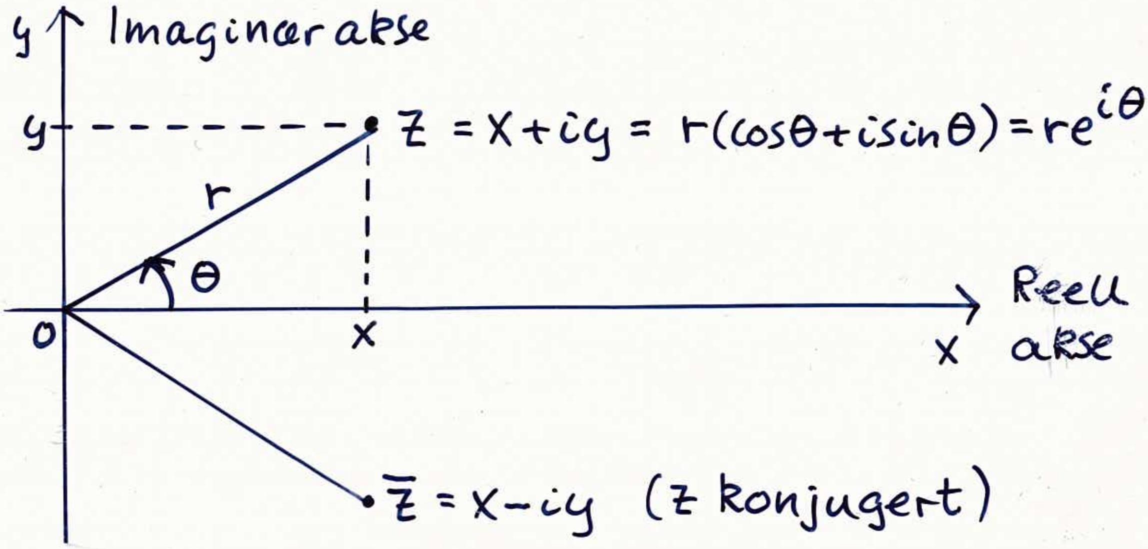
Komplekse tall (K 13.1 - 13.2)

$$z = x + iy \quad = r(\cos\theta + i\sin\theta) \quad = re^{i\theta}$$

normalform — polarform



$x = \operatorname{Re} z \quad = r\cos\theta$ — realdelen til z

$y = \operatorname{Im} z \quad = r\sin\theta$ — imaginærdelen til z

$r = |z| = \sqrt{x^2+y^2} = \sqrt{z\bar{z}}$ — absoluttverdien til z, avstanden fra z til 0

$\theta = \arg z, \; -\pi < \operatorname{Arg} z \leq \pi$ — argumentet til z, retningsvinkelen til z i forhold til positiv x-akse

$\tan\theta = \frac{y}{x}$

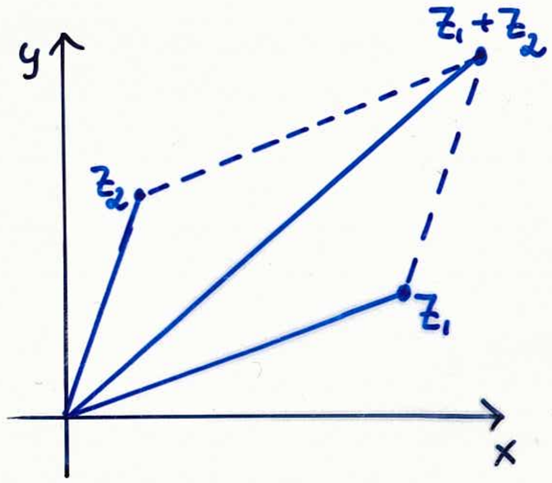
hovedverdien

$$\operatorname{Arg} z = \begin{cases} \arctan(y/x) & \text{for } z \text{ i 1. og 4. kvadrant} \\ \pi + \arctan(y/x) & \text{for } z \text{ i 2. kvadrant} \\ -\pi + \arctan(y/x) & \text{for } z \text{ i 3. kvadrant} \end{cases}$$

Regning med komplekse tall $(z_1 = x_1 + iy_1,\ z_2 = x_2 + iy_2)$

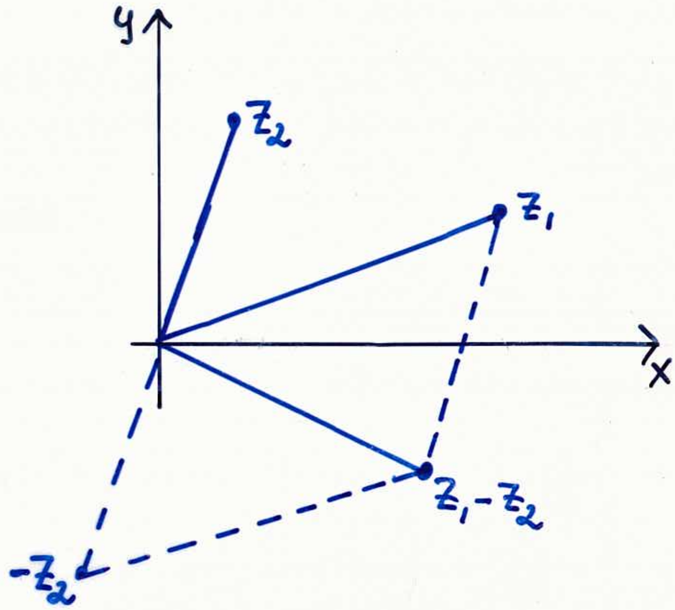
$$z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$$

$$z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$$



$|z_1 + z_2| \leq |z_1| + |z_2|$

(trekantulikheten)



$|z_1 - z_2|$ = avstanden mellom z_1 og z_2

$$z_1 \cdot z_2 = (x_1 + iy_1)(x_2 + iy_2)$$

$$= (x_1x_2 - y_1y_2) + i(x_1y_2 + x_2y_1) \qquad (i^2 = -1)$$

$$\frac{z_1}{z_2} = \frac{x_1 + iy_1}{x_2 + iy_2} = \frac{(x_1 + iy_1)(x_2 - iy_2)}{(x_2 + iy_2)(x_2 - iy_2)}$$

$$= \frac{(x_1x_2 + y_1y_2) + i(x_2y_1 - x_1y_2)}{x_2^2 + y_2^2}$$

$$e^t = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \frac{t^5}{5!} + \cdots$$

$t = i\theta$:

$$e^{i\theta} = 1 + i\theta + \frac{(i\theta)^2}{2!} + \frac{(i\theta)^3}{3!} + \frac{(i\theta)^4}{4!} + \frac{(i\theta)^5}{5!} + \cdots$$

$$i^2 = -1 \quad i^3 = -i \quad i^4 = 1 \quad i^5 = i \quad \ldots$$

$$e^{i\theta} = \underbrace{\left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - + \cdots\right)}_{\cos\theta} + i\underbrace{\left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - + \cdots\right)}_{\sin\theta}$$

NB! $\boxed{e^{i\theta} = \cos\theta + i\sin\theta}$ Eulers formel (K s.58)

$\theta = 1$: $e^i = \cos 1 + i \sin 1 = 0.5403 + 0.8415i$

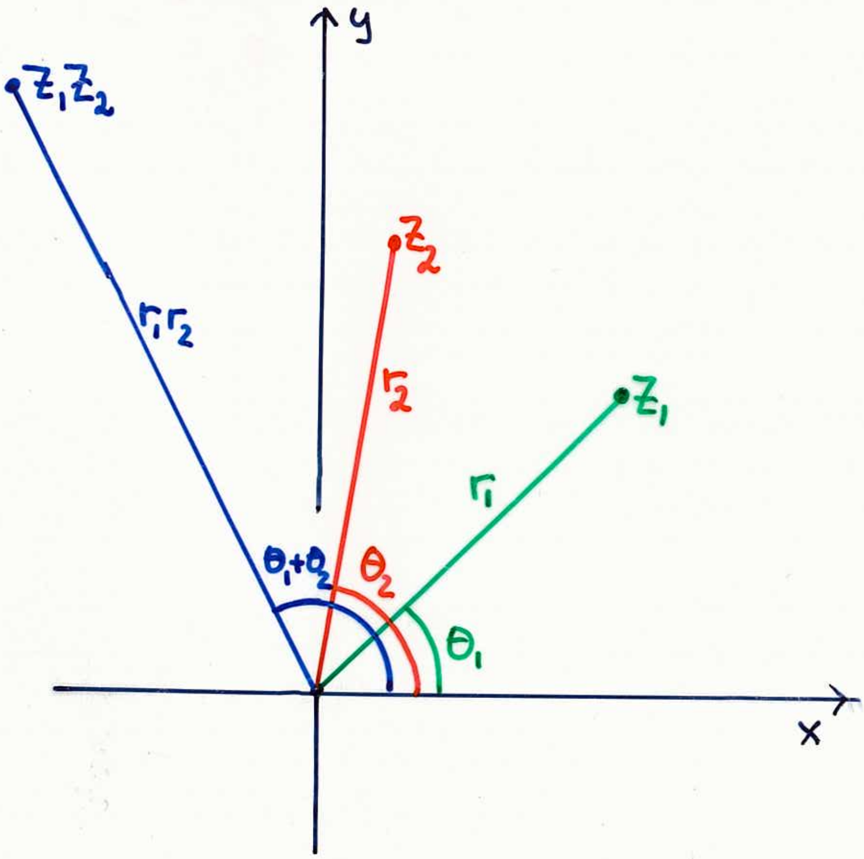
$\theta = \pi$: $e^{i\pi} = \cos\pi + i\sin\pi = -1$

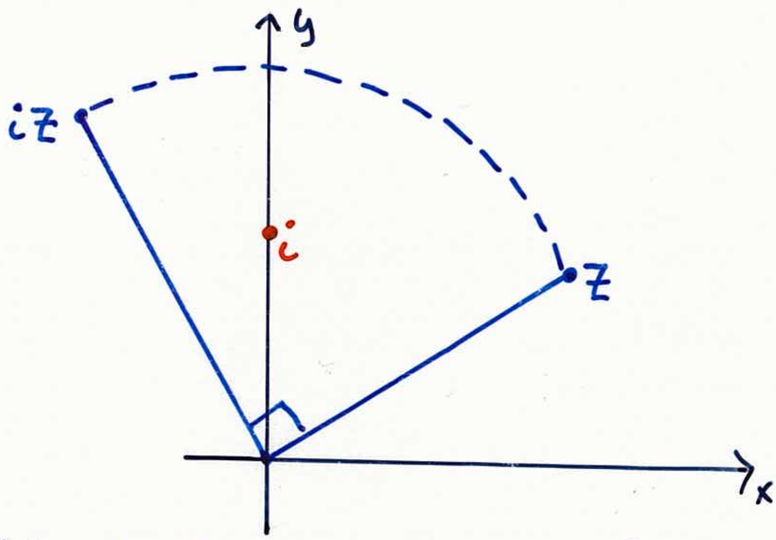
Multiplikasjon og divisjon på polarform

Gitt $z_1 = r_1 e^{i\theta_1}$ og $z_2 = r_2 e^{i\theta_2}$

$$\boxed{z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)} \qquad |z_1 z_2| = |z_1||z_2| \qquad \arg(z_1 z_2) = \arg z_1 + \arg z_2}$$

$$\boxed{\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)} \qquad \left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|} \qquad \arg\frac{z_1}{z_2} = \arg z_1 - \arg z_2}$$





Multiplikasjon med i