

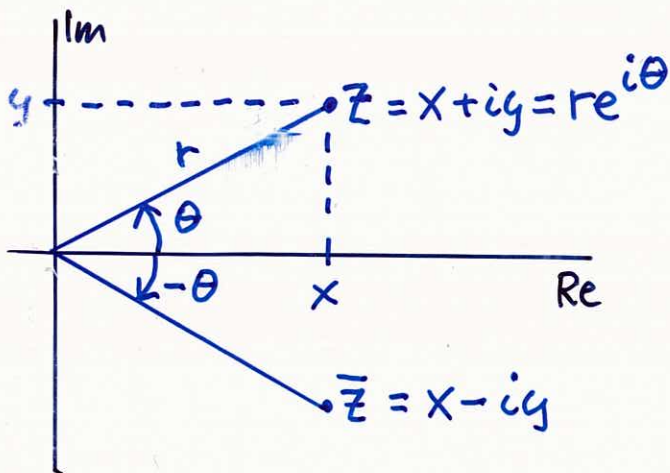
# Komplekse tall (K13.1, 13.2 + side 57-58)

$$z = x + iy = r(\cos\theta + i\sin\theta) = re^{i\theta}$$

normalform

polarform

$$e^{i\theta} = \cos\theta + i\sin\theta \text{ (Eulers formel)}$$



$$x = \operatorname{Re} z \quad x = r\cos\theta$$

$$y = \operatorname{Im} z \quad y = r\sin\theta$$

$$r = |z| \quad r = \sqrt{x^2 + y^2}$$

$$\theta = \operatorname{arg} z \quad \tan\theta = y/x \quad *$$

$$-\pi < \operatorname{Arg} z \leq \pi$$

$$*) \begin{cases} \theta = \arctan \frac{y}{x} & \text{for } z \text{ i 1. og 4. kvadrant } (-\frac{\pi}{2} < \theta < \frac{\pi}{2}) \\ \theta = \arctan \frac{y}{x} + \pi & \text{for } z \text{ i 2. og 3. kvadrant } (\frac{\pi}{2} < \theta < \frac{3\pi}{2}) \end{cases}$$

	normalform	polarform
$z_1 = z_2$	$x_1 = x_2$ og $y_1 = y_2$	$r_1 = r_2$ og $\theta_1 = \theta_2 + 2k\pi$ ( $k=0, \pm 1, \pm 2, \dots$ )
$i^2 = -1$ $z_1 \cdot z_2$	$(x_1 + iy_1)(x_2 + iy_2) = \dots$	$r_1 r_2 e^{i(\theta_1 + \theta_2)}$
$\frac{z_1}{z_2}$	$\frac{z_1 \bar{z}_2}{z_2 \bar{z}_2} = \frac{z_1 \bar{z}_2}{ z_2 ^2} = \dots$	$\frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}$

n-te røtter :  $\omega = \sqrt[n]{z}$

$$\omega^n = z$$

$$R^n e^{in\phi} = r e^{i\theta}$$

$$R^n = r \text{ og } n\phi = \theta + 2k\pi$$

$$R = \sqrt[n]{r} \text{ og } \phi = \frac{\theta + 2k\pi}{n}$$

$$\omega_k = \sqrt[n]{r} e^{i(\theta + 2k\pi)/n}, \quad k = 0, 1, 2, \dots, n-1$$

Komplekse løsninger i  $y'' + ay' + by = 0$

$$e^{(\alpha + i\beta)x} = e^{\alpha x + i\beta x} = e^{\alpha x} (\cos \beta x + i \sin \beta x)$$

$$e^{(\alpha - i\beta)x} = e^{\alpha x - i\beta x} = e^{\alpha x} (\cos \beta x - i \sin \beta x)$$

$$\text{Sum : } 2e^{\alpha x} \cos \beta x$$

$$\text{Diff. : } 2ie^{\alpha x} \sin \beta x$$

$$e^{\alpha x} \cos \beta x$$

$$e^{\alpha x} \sin \beta x$$

Reelle

løsninger