

Complex Numbers in Polar Form

$$z = x + iy, \quad r = |z| = \sqrt{x^2 + y^2},$$

$$x = r \cos \theta, \quad y = r \sin \theta,$$

$\theta = \arctan(y/x) + k\pi$ according to the quadrant

$$z = r(\cos \theta + i \sin \theta) = re^{i\theta}.$$

Multiplication and Division in Polar Form

$$z_1 = r_1 e^{i\theta_1}, \quad z_2 = r_2 e^{i\theta_2}$$

$$z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}, \quad \frac{z_1}{z_2} = \frac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}.$$

De Moivre's Formula

$$z^n = (r(\cos \theta + i \sin \theta))^n = r^n (\cos n\theta + i \sin n\theta)$$

Equation $w^n = z = r(\cos \theta + i \sin \theta)$ has n solutions

$$w_k = \sqrt[n]{r}(\cos(\theta + 2pk)/n + i \sin(\theta + 2pk)/n), \\ k = 0, 1, \dots, n - 1$$

Points w_0, w_1, \dots, w_{n-1} lie on a circle of radius $\sqrt[n]{r}$ and center 0 and constitute the vertices of a regular polygon.

Quadratic Equation $z^2 + pz + q = 0$ has two (complex) solutions

$$z_{1,2} = (-p \pm \sqrt{p^2 - 4q})/2$$