Second Order Linear Differential Equations

$$y'' + p(x)y' + q(x)y = r(x)$$

The equation is called homogeneous if $r(x) \equiv 0$ and nonhomogeneous if $r(x) \neq 0$.

Superposition principle for homogeneous equations

If y_1 and y_2 are solutions to the equation

$$y'' + p(x)y' + q(x)y = 0, \qquad (*)$$

then so is $c_1y_1 + c_2y_2$.

A pair of linearly independent solutions of (*) is called a basis of solutions and in this case $c_1y_1 + c_2y_2$ is called the general solution of (*).

Linearly independent functions

Two functions y_1 and y_2 are called linearly independent on the given interval I if the identity

$$k_1 y_1(x) + k_2 y_2(x) \equiv 0$$
 on *I*

implies $k_1 = k_2 = 0$.

Two functions that are not linearly dependent are called linearly independent.

In other words, y_1 and y_2 are linearly dependent if $y_2 \equiv 0$ or y_1/y_2 is constant on I.