Second Order Linear Homogeneous Equations with Constant Coefficients

$$y'' + ay' + by = 0$$

1. Write the characteristic equation

$$\lambda^2 + a\lambda + b = 0.$$

2. Find the roots

$$\lambda_1 = \frac{1}{2}(-a + \sqrt{a^2 - 4b}), \ \lambda_2 = \frac{1}{2}(-a + \sqrt{a^2 - 4b}).$$

and see what is the case:

Case I $a^2 - 4b > 0$, two real roots,

Case II $a^2 - 4b = 0$, a double real root,

Case III $a^2 - 4b < 0$, two complex conjugate roots.

3. Find a general solution

   Case I $y(x) = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$;

   Case II $y(x) = (c_1 + c_2 x)e^{-ax/2}$;

   Case III $y(x) = (c_1 \cos \omega x + c_2 \sin \omega x)e^{-ax/2}$, where $\lambda_1 = -a/2 + \omega i,\ \lambda_2 = -a/2 - \omega i$.

4. Given an initial value problem

$$y(x_0) = K_0,\ y'(x_0) = K_1,$$

   find a particular solution by determining the constants c_1 and c_2.