

Second Order Linear Differential Equations: Reduction of order

$$y'' + p(x)y' + q(x)y = 0$$

Assume that we know one solution y_1 of the homogeneous equation.

Then we can find another one.

We write $y_2 = uy_1$, then

$$u'' + u' \frac{2y_1' + py_1}{y_1} = 0$$

and

$$u = \int U(x)dx, \quad \text{where} \quad U = \frac{1}{y_1^2} e^{-\int p(x)dx}.$$

Finally, $y_2 = y_1 \int U(x)dx$.

Solutions y_1 and y_2 are linearly independent.