

## Mass spring system: Free oscillation

Undamped system  $my'' + ky = 0$

$$y(t) = A \cos \omega_0 t + B \sin \omega_0 t = C \cos(\omega_0 t - \delta),$$
$$\omega_0 = \sqrt{\frac{k}{m}}, \quad C = \sqrt{A^2 + B^2}, \quad \tan \delta = \frac{B}{A}.$$

$y(t)$  is called a **harmonic oscillation**,  
it describes periodic motion with **period**  $2\pi/\omega_0$ ,  
**frequency**  $\omega_0/2\pi$  and **amplitude**  $C$ .

Initial values are  $y(0) = A$ ,  $y'(0) = \omega_0 B$ .

Damped system  $my'' + cy' + ky = 0$ .

Roots of the characteristic equation are

$$\lambda_{1,2} = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m}$$

- $c^2 > 4mk$ , two real roots, **overdamping**
- $c^2 = 4mk$ , double real root, **critical damping**
- $c^2 < 4mk$ , two complex roots, **underdamping**