Second Order Linear Homogeneous Equations Existence and Uniqueness.

$$y'' + p(x)y' + q(x)y = 0 \qquad (*)$$

$p(x)$ and $q(x)$ are continuous on an interval I.

Theorem 1 If x_0 is a point on I then the initial value problem $y(x_0) = K_0,\ y'(x_0) = K_1$ for equation (*) has a unique solution.

The Wronski determinant of two functions y_1 and y_2 is a function defined by

$$W(y_1, y_2) = y_1 y_2' - y_2 y_1'.$$

Theorem 2 If y_1 and y_2 are two solutions to (*) on I. Then the following conditions are equivalent:
1. y_1 and y_2 are linearly independent.
2. $W(y_1, y_2)(x_0) \neq 0$ for any x_0 on I.
3. $W(y_1, y_2) \not\equiv 0$ on I.

Theorem 3 If y_1 and y_2 are two linearly independent solutions to (*) on I. Then any solution of (*) an I can be written as $c_1 y_1 + c_2 y_2$.