Method of Undetermined Coefficients

$$y'' + ay' + by = r(x)$$

We have linear equation with constant coefficients and special right hand side. In order to find a solution, look at $r(x)$ and find appropriate coefficients in the expression for $y(x)$ below.

$r(x)$	$y(x)$
$ke^{\lambda x}$	$Ce^{\lambda x}$, if λ is not a root of $\lambda^2+a\lambda+b=0$; $xCe^{\lambda x}$, if λ is one of the roots of $\lambda^2+a\lambda+b=0$; $x^2Ce^{\lambda x}$, if λ is a double root of $\lambda^2+a\lambda+b=0$.
$ke^{\alpha x}\cos\omega x$, $ke^{\alpha x}\sin\omega x$	$e^{\alpha x}(K\cos\omega x + M\sin\omega x)$, if $\alpha+i\omega$ is not a root of $\lambda^2+a\lambda+b=0$; $xe^{\alpha x}(K\cos\omega x + M\sin\omega x)$, if $\alpha+i\omega$ is a root of $\lambda^2+a\lambda+b=0$.
kx^n	$K_nx^n + K_{n-1}x^{n-1} + \ldots + K_1x + K_0$, if $b\neq 0$; $x(K_nx^n + K_{n-1}x^{n-1} + \ldots + K_1x + K_0)$, if $b=0$, $a\neq 0$; $x^2(K_nx^n + K_{n-1}x^{n-1} + \ldots + K_1x + K_0)$, if $b=0$, $a=0$.
$kx^ne^{\lambda x}$	$(K_nx^n + \ldots + K_1x + K_0)e^{\lambda x}$, if λ is not a root of $\lambda^2+a\lambda+b=0$; $x(K_nx^n + \ldots + K_1x + K_0)e^{\lambda x}$, if λ is one of the roots of $\lambda^2+a\lambda+b=0$; $x^2(K_nx^n + \ldots + K_1x + K_0)e^{\lambda x}$, if λ is a double root of $\lambda^2+a\lambda+b=0$.

If $r(x)$ is a linear combination of the above functions take $y(x)$ to be a sum of the corresponding expressions.