## Mass spring system: Forced oscillations

Undamped system $my'' + ky = F_0 \cos \omega t$

$\omega \neq \omega_0$, where $\omega_0 = \sqrt{\frac{k}{m}}$,

$$y(t) = y_h + y_p = C \cos(\omega_0 t - \delta) + \frac{F_0}{m(\omega_0^2 - \omega^2)} \cos \omega t,$$

Resonance: $\omega = \omega_0$,

$$y(t) = y_h + y_p = C \cos(\omega_0 t - \delta) + \frac{F_0}{2m\omega_0} t \sin \omega_0 t,$$

Damped system $my'' + cy' + ky = F_0 \cos \omega t$.

The transient solution (general solution of the non-homogeneous system) is

$$y(t) = y_h(t) + y_p(t).$$

Each solution of the homogeneous equation goes to zero when $t \to \infty$. Thus the transient solution approaches the steady-state solution

$$y_p = C(\omega) \cos(\omega t - \eta).$$